EXCLUSION, COMPETITION, AND REGULATION IN THE RETAIL LOAN MARKET

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Abstract
We analyze an imperfectly-competitive market for loans where loan-making institutions set both interest rates and minimum credit quality requirements necessary to be eligible for a loan. We analyze a monopoly lending financial institution and then the impact of entry of another credit provider into a monopolistic market, by computing market shares, the number of excluded (potential) borrowers, and social welfare. We also analyze the effect of regulating interest rates, and demonstrate that this policy reduces social welfare. Some borrowers may gain from a lower mandated interest rate, but others may become excluded from the loan market.

Keywords: Credit quality, financial regulation, lending arrangements.
JEL Classification Numbers: D43, G21
1 Introduction

Market power in the banking retail sector can create price distortions and market segmentation. High loan rates can block many small borrowers from access to loans. The degree of competition is thus important for credit allocation. Our objective, in this article, is to examine the retail banking market and analyze how it is divided into different segments. In particular we wish to learn how the phenomenon of exclusion is created and what are the factors that influence its magnitude. We argue that loan market power can give rise to the exclusion of borrowers.

The problem of exclusion of retail costumers has been largely overlooked by the literature, perhaps because it is hard to test it empirically. Surveys by the FDIC (2009, 2012), in the USA found that access to standard banking services is not universal. The 2009 survey indicate that 7.3 percent of households do not have checking accounts and 17.9 percent have checking account but use alternative financial service providers for borrowing and even check cashing. Financial exclusion is also a problem in Germany. According to ZEWnews of October 2012, in Germany, only 76 percent of households have a standard bank account that allows customers to overdraft. In the UK, the study by FIT (2009) indicates that 5 percent of the adult population does not have access to a transaction account.

In the following analysis we rely on the institutional set-up of the retail loan market. Loan markets are often characterized by borrowers who have to meet certain minimum basic quality requirements. This is the typical situation in many consumer loan markets where the lending bank determines not only the interest rate on loans, but also some minimum credit rating that is often reported by credit agencies. Berger and Udell (1995) report that similar minimum quality requirements exist in the market for small business loans. In this paper we analyze an imperfectly-competitive market for loans where the banks set both interest rate and minimum credit quality requirements that potential borrowers should possess in order to be eligible for a loan. We investigate the impact of competition on the market for small borrowers. We also analyze the effect of interest regulation on social welfare.

We use a simple model in which banks choose how much to lend, given the quality of loans that can be added to their portfolios. Of particular interest to us is the impact of inter-bank com-
petition. We model, specifically, the nature of competition between two competing banks. The results help to identify the quality-cost tradeoff. And it’s impact on market share and on the degree of exclusion. We also show how the competitive results are influenced by regulation. The background for this analysis is the competitive environment that banks are facing in the retail market.

Banks also face increased competition from non-banks financial intermediary. This is more applicable to business lending. In the retail segments, however it is appropriate to focus on the competition within the banking industry. We describe the competition for market shares when the customer quality is an important consideration. Specifically, we investigate how the profitability of each bank changes as a response to competition for market shares and the degree of exclusion. Since the banking sector is heavily regulated, we also investigate the impact of price regulation on the equilibrium outcome in the industry.

We focus on retail banking where the borrowers are small and negotiations about loan contracts are very costly. In commercial lending contract negotiations are possible and are likely to produce adjustments of interest rates that reflect the unique risk profile of the borrower. In contrast, when borrowers are small banks tend to aggregate costumers into quality groups and set a fixed loan rate to match the quality profile of the group.

There is a large literature on the commercial loan market but the literature on consumer lending is not as vast. Historically, much of the empirical research in that area focused on issues of credit scoring and on the possibility of rationing and exclusion, see Heuson, Passmore, and Sparks (2001). A few outstanding articles such as Calem and Mester (1995) focused on the differential interest rates that exist in the consumer loan market. In our model we focus on competition between lenders in a loan market that has a similar structure. Other contributions explain the deterioration in credit quality that is mentioned, for example, by Black and Morgan (1999). They report that in the early 1990’s there was an increase in the share of credit cards held by low-income persons and a parallel increase in the debt repayments-to-income ratio. The results of our model are in line with their conclusion that, as competition increases lenders (e.g. card issuers) expand sales to less credit worthy costumers.
Our results have some implications that go beyond banking. A few years ago, Ross (2012) pointed out that the finance literature does not address properly the marketing of financial products. The role of competitive behavior of financial firms has not received sufficient attention. In this paper we attempt to fill part of the void and to demonstrate the impact of competitive behavior on the size of the market. In particular our model shows that increased competition among firms may lead to lower prices via the standard margin undercutting argument. However, increased competition may also change customer quality requirements when firms compete for market share. In this way, quality provides another dimension on which to compete.

The paper is organized as follows. Section 2 surveys some related literature. Section 3 constructs the benchmark model of a single loan-making institution facing differentiated credit-rated consumers. Section 4 extends the environment to an imperfectly-competitive loan making industry by analyzing entry. Section 5 analyzes the welfare consequences of the imposition of a market-uniform interest rate for all borrowers. Section 6 analyzes a two-stage simultaneous moves environments and lists the differences and similarities with the sequential moves model. Section 7 concludes with further discussions.

2 Related Literature

Academic researchers in the banking area generally adapted the principal-agent problem to fit specific financial situations. The typical framework includes an owner of a firm who intends to invest in a real project. She needs to obtain a loan in order to execute her plans. The bank has the funds but considers itself as the weak side of the deal. This in turn leads to an inherent conflict of interests between the borrower (the agent) with a real investment plan that needs the financing and the banker (the principal) that has the funds.

According to this version of the principal-agent problem there are three methods that the banker may use in order to mitigate the conflict. First, she can collect information about the entrepreneur before funding the project. As noted by Laffont and N’Guessan (2000), the screening out of bad projects entails considerable costs. Second, an elaborate contract between the two parties can allocate the benefits of the real investment (i.e. cash flow, control and termination rights)
between the two parties so as to provide incentives to the borrowers while protecting the interest of the lending institution. Rosen (1982) noted that writing comprehensive contracts is also expensive. Third, while the contract is in force the banker can monitor the entrepreneur in order to ascertain the real project is run in the proper way. This, of course, requires additional costs. Because of the high costs, most of the analysis since the pioneering work of Kihlstrom and Laffont (1979) considered the relations between two firms: a bank and a corporate borrower. The size of the loan would justify some of the extra costs that banks usually incur.

The academic literature that deals with competition in the financial sector can be divided into two broad areas: competition between banks and other intermediaries and competition within the banking sector. In the competition with other lenders, the banks are grouped into one industry, which is then compared with another industry (e.g. the brokerage industry). A common comparison is between banks and so-called arm’s length investors. For example, some studies assume that banks can re-organize firms in stressful circumstances more efficiently than arm’s length investors. On the other side, non-bank financial investor possess a lower opportunity cost of capital than banks. The endogenous outcome, mentioned by Rajan (2012), is that different lenders dominate in different segments of the asset quality range. High quality firms prefer the bond market in order to reduce their finance costs. At the other end of the quality spectrum, poor quality firms are likely to borrow from banks because they might need to use their organizing skills.

A related view regards banks as skillful project screeners. In models such as Diamond (1991) and Holmstrom and Tirole (1997), market segmentation is created as a solution to incentive problems. Companies without incentive problems will borrow from arm’s length investors. On the other hand, firms with serious incentive problems will use banks because their screening (or monitoring) services will raise the financial value of such borrowers.

Market segmentation between two providers of small loans is also explored in the growing literature on the nature of competition between lenders in developing countries. In many countries “informal loans” co-exist alongside regular bank loans. Such loans are provided by credit cooperatives, non-bank moneylenders and traders. According to Germidis, Kessler, and Meghir (1991), and Hoff and Stiglitz (1997) such lenders account for more than 50% of the small loan mar-
kets in several Asian countries. The result is a segmented market for such loans. The institutional structure of retail credit in developing countries has been explored also by Bell, Srintvasan, and Udry (1997) and Jain (1999). They model informal lenders as competitors to conventional banks. We consider a similar structure of two competitors in the retail credit market. The driving force of the present model is the interplay between two different constrains that lenders face: price and quality. We also allow for the possibility of credit rationing even under monopolistic competition without asymmetric information.

The other main branch of the literature concentrates on competition within the banking sector. A way to categorize the literature is offered by Boot and Thakor (2002). When they survey the literature on oligopolistic rent generation, they find three relevant branches used for analyzing bank competition. These are spatial models, models of monopolistic competition and ex-post generations of rent by the creation of banking relationships. This last form of competition received a lot of attention recently.

An important branch of the financial literature that analyzes competition within the banking sector deals with the implications of borrower-bank relationship to bank profitability. Petersen and Rajan (2012); Boot and Thakor (2002), and Longhofer and Santos (2000), among others, noted that the propriety information produced due to the relationship between a bank and a borrower can create monopoly rents for the bank. Eventually, however, other banks tend to reduce these rents by competition. Theses models focus on the impact of the resulting inter-temporal pattern of loan prices.

Our paper differs from other contributions about the behavior of financial intermediaries in three ways: First, unlike many important contributions, we are not concerned with loan contact design. We focus on how markets are shared between suppliers of credit when the contracts structure is simple and the customers have different measures of quality. Second, most of the literature focuses on ex-post market segmentation. Many papers, such as Melnik and Plaut (1998) analyze shifts in market segmentation without addressing the competitive decision of rival suppliers of

\footnote{The interaction between the bank and the borrower have also been discussed in the literature that deals with the design of financial systems and financial contracts. Instructive examples include Berlin and Mester (1992), Gompers (2012), and Thakor (1996).}
credit. The competitive decisions of credit providers are the central element of our model. Third, we focus on loans to retail customers and not on lending relationship with borrowing companies.

In a broad sense, our paper is linked to the literature that relates profit levels to quality differentiation in oligopolistic markets. This literature can be traced back to Shaked and Sutton (1982) who investigate why two different qualities are offered by two rival firms and both can still earn a profit. In their analysis, customers possess different quality preferences that depend on their initial endowments. Similar models in that literature show how the profitability of firms, in an oligopolistic setting, is linked to the quality attributes of the product that they sell. However, Shaked and Sutton (1982) do not consider oligopolistic competition of the type that we introduce here.\(^2\) We use a different approach. In the following model, borrowers (customers) have identical preferences but possess heterogeneous credit risk (quality levels). We focus not the variety of products to be offered in an equilibrium, but rather on the choice of price and quality pairs made by competing banks.

### 3 A Single Loan-Making Institution

Consider a market where a single loan-making institution, acting as a monopoly, is facing a continuum of potential borrowers. The borrowers are indexed by a quality parameter \(\rho\) on the interval \([0, 1]\) with a uniform density of \(n\) borrowers per type. Therefore, the total number of potential borrowers equals to \(n\).

Following Lazear (1986), the market for loans is such that the retail prices (interest rates) tend to be uniform across lenders within a given period and that there is very little or no haggling. This is also the observed behavior in the banking sector. Lazear (1986) noted that in most retail markets, the seller agrees implicitly to sell to any person who is willing to pay the posted price. Ordinarily, no price bargaining occurs in such markets. Prices may change over time, but at any given period they are roughly the same for all customers. Lazear (1986) reports that are two reasons why a seller, when faced with many potential customers, would choose a strict retail pricing over some form of haggling. First, haggling may encourage strategic behavior on the part of the customers.

\(^2\)Our model is also related to the literature that analyzes lenders incentives to screen borrowers in different market structures such as mentioned by Hauswald and Marquez (2003).
Such behavior is absent when a standard retail price is used. Second, haggling is costly to the owners of the firm, because it requires delegation of authority (permission to offer discounts) to employees. In order to eliminate adverse selection of employees, the firm may decide to announce a rigid price each period. Haggling is likely to occur when the owner, who sets the price, is also the sales agent in the firm. This is not the type of market considered in this paper.

3.1 Borrowers

There is a continuum of \( n > 0 \) potential borrowers who are uniformly distributed on the interval \([0, 1]\). Each potential borrower is indexed by \( \rho \in [0, 1] \) to be interpreted below. Each borrower wishes to borrow $1 in order to buy an income-producing asset (investment). The return on this investment is random, so that the return on a one-dollar investment by a type \( \rho \) borrower is

\[
R_\rho \overset{\text{def}}{=} \begin{cases} 
V & \text{with probability } \rho \\
0 & \text{with probability } 1 - \rho.
\end{cases}
\]  

The parameter \( V > 1 \) is the gross return on $1 investment if it does not fail. If it fails, the entire invested amount is lost. Thus, the expected gross return is \( \rho V \), which means that borrowers indexed by higher values of \( \rho \) have higher expected returns on this investment.

For the sake of simplicity, potential borrowers do not have any funds of their own.\(^3\) Therefore, the only way in which they can finance their investment is by borrowing. Let \( r \) denote the endogenously-determined nominal interest rate, set by the lending institution. Utility of a borrower, of type \( \rho \), depends on the income from the individual’s project net of the cost of the loan. We define the utility of a type \( \rho \) borrower (\( 0 \leq \rho \leq 1 \)) by

\[
U_\rho \overset{\text{def}}{=} \begin{cases} 
V - (1 + r) & \text{with probability } \rho \\
0 & \text{with probability } 1 - \rho.
\end{cases}
\]  

The utility function (2) implies that a type \( \rho \) borrower is bankrupt with probability \( 1 - \rho \), in which case the borrower must default on his loan by not paying back to the bank the interest and the

\(^3\)In our model, borrowers do not have any other assets as well. Unlike Manove, Padilla, and Pagano (2001), we do not consider the role of collateral. In their model of “lazy banks” high-quality borrowers post collateral in competitive market in order to distinguish themselves from low-quality borrowers. Therefore, banks can forgo monitoring because borrowers’ type is revealed via their collaterals.
principal on this loan. We will therefore refer to a borrower’s index number ρ as a borrower’s quality parameter, as this parameter reflects the probability of paying back a loan with interest. Alternatively, we will also refer to ρ as a borrower’s credit rating. We assume that this credit rating is public knowledge because quality information about potential borrowers can be obtained at a low cost from credit rating institutions.

3.2 Loan-making institution

We now model a risk-neutral profit-maximizing loan-making institution, which we call the lender. The lender has sufficient funds to lend to each of the n potential borrowers. The lender determines the interest rate, r, and sets a minimum credit rating for loan eligibility, to be denoted by ˆρ. Essentially, each borrower has to demonstrate a minimum quality level to be eligible for receiving a loan. Since borrowers are arranged according to their probability of paying back their loans, the debt market is partitioned into ineligible (excluded) and eligible borrowers, as illustrated in Figure 1.

![Figure 1: Excluded and eligible borrowers.](image)

In order to define the lender’s profit function consider an arbitrary type ρ borrower who is eligible for a $1 loan. The lender will then collect an amount of 1 + r (principal plus interest) with probability ρ for the $1 loan given to a borrower. Thus, the lender chooses, both ˆρ and r to maximize expected profit given by

$$\max_{\hat{\rho}, r} \pi^m = n(1 + r) \int_{\hat{\rho}}^1 \rho \, d\rho - n(1 - \hat{\rho}).$$  (3)

The first term is the expected gross return collected from all eligible borrowers; i.e., borrowers indexed by ˆρ ≤ ρ ≤ 1. The second term is the total amount of money that the lender lends to all
borrowers whom he finds to be eligible.

3.3 Equilibrium minimum credit rating and interest rate

Inspecting borrowers’ utility functions given in (2) reveals that a potential borrower benefits from taking a loan if

\[ \rho [V - (1 + r)] \geq (1 - \rho)0, \text{ or } r \leq V - 1 \]

which is independent of \(\rho\) (the borrower’s type). Therefore, by setting \(r^m = V - 1\) the lender ensures that all potential borrowers benefit from submitting loan applications. Substituting the profit-maximizing interest rate \(r = V - 1\) into (3), the lender then left to choose a minimum credit rating for loan eligibility, \(\rho^m\), that solves

\[
\max_{\rho} \pi^m = nV \int_{\rho}^{1} \rho \, d\rho - n(1 - \rho) = \frac{n}{2} [V(1 - \rho^2) - \rho^2 + 2\rho - 1],
\]

which is the total return on a portfolio of loans to all eligible borrowers. The first- and second-order conditions for a maximum are

\[
0 = \frac{\partial \pi^m}{\partial \rho} = n(1 - V \rho^m), \quad \text{and} \quad \frac{\partial^2 \pi^m}{\partial \rho^2} = -nV < 0.
\]

Therefore, the monopoly lender’s profit-maximizing interest rate, minimum credit rating eligible for a loan, and the resulting expected profit are given by

\[
r^m = V - 1, \quad \rho^m = \frac{1}{V}, \quad \text{and} \quad \pi^m = \frac{n(V - 1)^2}{2V}.
\]

Observe that the reason why the monopoly lender limits the qualified credit ratings to \(\rho \geq 1/V\) is that the investments made by these borrowers bear nonnegative returns since \(\rho V - 1 \geq \rho^m V - 1 = 0\).

Figure 2 shows the expected surplus of each borrower \(\rho\) under the equilibrium interest rate and minimum eligibility credit rating given in (5).

Figure 2 shows that borrowers’ surplus \((\rho V - 1)\) increases with the borrower’s quality index \(\rho\), because \(\rho\) is also the probability that the investment yields a return \(V\). Note that the negative surplus in Figure 2 is shown only for the sake of illustration because the lender refuses loans to all borrowers indexed by \(\rho < \hat{\rho}\).
When the lender chooses the interest rate $r^m$ given in (5), the lender’s expected profit from lending to a borrower type $\rho$ is $\rho(1 + r^m) - 1$. That is, by choosing $r^m = V - 1$, this monopoly lender can extract all borrower surplus. Therefore, expected profit equals aggregate borrower surplus which equals to the area of the lower right triangle in Figure 2. Thus, our assumption that the interest rate charged to all borrowers is the same turns out to be not restrictive. This is so because, when the a borrower’s investment yields a return, all borrowers receive the same return $V$ on their investment although the investment success probability varies across borrower types.

We can draw the following implications from the equilibrium values given in (5):

**Result 1.** An increase in borrowers’ return on their investment, $V$, will increase (a) the lending interest rate, $r^m$, (b) the number of eligible borrowers, $1 - \rho^m$, and (c) the lender’s profit level, $\pi^m$.

Result 1 can be demonstrated using the following example: Equation (5) implies that the lender will make loans to half of the potential borrowers ($\rho^m = 1/2$) when the stochastic return equals twice the $1$ investment level ($V = 2$). Similarly, if $V = 3$, $\rho^m = 1/3$ meaning that the lender makes loans to $2/3$ of the potential borrowers and denies loans to $1/3$ of them.\textsuperscript{4} If we interpret $V$ as a

\textsuperscript{4}Fluctuations in lending due to changes in economic conditions confirm this type of behavior. Oliner and Rudebusch (1996) argue that banks do not uniformly call in loans during contractions, but lenders uniformly flee to high quality borrowers when they lend during recessions. Their flight to quality argument is based on the assumption that there is a stronger impact on the choice of loans during recessions than in booms. Similar results have been found by Lang and Nakamura (1995).
measure of the condition of the economy, then this example shows that the market expands as the economy improves. In other words, ever potential borrowers are excluded when the return on private investment increases.

### 3.4 Social optimum

To determine the minimum credit rating that maximizes social welfare, we define the social welfare function as the sum of borrowers’ utilities and industry profit, given by (2) and (3), respectively. Formally,

\[
W = \int_{\rho^*}^{\frac{1}{V}} U_{\rho} \, d\rho + \pi^m
\]

\[
= n \int_{\rho^*}^{\frac{1}{V}} \rho [V - (1 + r)] \, d\rho + n(1 + r) \int_{\rho^*}^{\frac{1}{V}} \rho \, d\rho - n(1 - \rho^*) = \int_{\rho^*}^{\frac{1}{V}} (\rho V - 1) \, d\rho.
\]

The interest rate \( r \) cancels out in the social welfare function since it is merely a transfer from borrowers to lenders. Maximizing (6) with respect to the socially-optimal minimum credit requirement yields

\[
\rho^* = \frac{1}{V} \quad \text{hence} \quad W^* = \frac{n(V - 1)^2}{2V},
\]

where \( W^* \) is the corresponding maximal social welfare level. Comparing (7) with (5) yields the result.

**Result 2.** A monopoly lending institution sets the socially-optimal minimum credit requirement. This lender extracts the entire surplus from borrowers.

Result 2 demonstrates an old idea attributed to Swan (1970a,b) where a monopoly firm that can freely set its price has no incentives to distort other choice variables such as quality and durability. In the present case, the monopoly lender is able to raise the interest to the monopoly level. Hence it has no incentive to choose any minimum credit requirement other than the socially-optimal level.
4 Duopoly Market for Loans

Consider now an entry of a second loan-making institution. Thus, we analyze a transition from a monopoly to a duopoly loan-making industry, where we denote by $A$ the incumbent institution, and by $B$ the entering institution. Note, however, that borrowers do not change their behavior. That is, each borrower takes at most one loan from one of the banks. Formally, in this section we analyze a sequential two-stage game. Section 6 analyzes a two-stage simultaneous moves environments and lists the differences and similarities with the sequential moves model analyzed in this section. During the first stage, lender $A$ simultaneously chooses the interest rate, $r_A$, and the minimum credit rating eligible for a loan, $\rho_A$. In the second stage, lender $B$ simultaneously chooses $r_B$ and $\rho_B$ taking $r_A$ and $\rho_A$ as given.

We look for a subgame-perfect equilibrium (SPE) for this game and therefore proceed by solving the game backwards.

4.1 Stage II: Lender $B$

Let $r_A$ and $\rho_A$ be given. The two options facing lender $B$ are:

(a) Sharing the market with lender $A$ by charging a higher interest than $A$ thereby serving the borrowers with a lower credit rating compared with the borrowers served by $A$.

(b) Undercutting lender $A$ by offering a lower interest than $A$, thereby leaving $A$ with no borrowers.

Figure 3 illustrates the two options available to lender $B$.

4.1.1 “Sharing” ($r_B \geq r_A$)

When lender $B$ sets $r_B \geq r_A$, Figure 3 (Top) illustrates that all borrowers indexed on $[\rho_A, 1]$ will borrow from $A$ (lowest interest on loans). Borrowers indexed on $[\rho_B, \rho_A]$ are forced to borrow from $B$ and would therefore pay a higher interest since their credit ratings make them ineligible for taking loans from lender $A$. Note that a large segment is still excluded from the market.

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5There is a third option available to lender $B$ which is to set its minimum credit rating requirement higher than that of lender $A$, that is $\rho_B \geq \rho_A$. However, as it turns out, this option is a special case of the “undercutting” option which is solved in equation (10).
Since all borrowers indexed by $\rho < \rho_A$ are not eligible for loans from $A$, in view of the utility function (2), lender $B$ raises the interest rate to the monopoly level $r_B = V - 1$. Thus, similar to (3), lender $B$ chooses $\rho^s_B$ ("$s$" stands for "sharing") that solves

$$\max_{\rho_B} \pi_B = nV \int_{\rho_B}^{\rho_A} \rho \ d\rho - n(\rho_A - \rho_B) = nV \left( \frac{\rho_A^2}{2} - \frac{\rho_B^2}{2} \right) - n(\rho_A - \rho_B).$$

The first- and second-order conditions for a maximum are

$$0 = \frac{\partial \pi_B}{\partial \rho_B} = n(1 - V \rho_B), \quad \text{and} \quad \frac{\partial^2 \pi_B}{\partial \rho_B^2} = -nV < 0.$$ 

Hence,

$$r^s_B = V - 1, \quad \rho^s_B = \frac{1}{V}, \quad \text{and} \quad \pi^s_B = \frac{n(V^2 - 2V \rho_A + 1)}{2V}.$$ 

where superscript "$s$" stands for sharing. Notice that, similar to the earlier case, comparing (9) and (5) reveals that in both market structures some credit rationing prevails.
4.1.2 “Undercutting” \((r_B < r_A)\)

When lender \(B\) sets \(r_B < r_A\), Figure 3 (Bottom) illustrates that all borrowers indexed on \([\rho_B, 1]\) will borrow from \(B\) and no one would borrow from \(A\) who charges a higher interest. Under \(r_B = r_A - \epsilon\), (where \(\epsilon\) is a small number) lender \(B\) chooses \(\rho_B^u\) that solves

\[
\max_{\rho_B} \pi_B \approx n(1 + r_A) \int_{\rho_B}^{1} \rho \, d\rho - n(1 - \rho_B) = \frac{n}{2} [r_A(1 - \rho_B^2) - \rho_B^2 + 2\rho_B - 1].
\]  

(10)

The first- and second-order condition for a maximum are given by

\[
0 = \frac{\partial \pi_B}{\partial \rho_B} = n(1 - \rho_B - r_A\rho_B), \quad \text{and} \quad \frac{\partial^2 \pi_B}{\partial \rho_B^2} = -n(1 + r_A) < 0.
\]

Hence,

\[
r_B^u \approx r_A, \quad \rho_B^u = \frac{1}{1 + r_A} \quad \text{and} \quad \pi_B^u = \frac{nr_A^2}{2(1 + r_A)},
\]  

(11)

where superscript “\(u\)” stands for undercutting.

4.1.3 Equilibrium strategy for lender \(B\)

We now determine the conditions under which lender \(B\) will find it more profitable to share the market with lender \(A\) rather than to undercut lender \(A\) and grab its entire market. Comparing (9) with (11) yields that sharing is profitable if

\[
\pi_B^s \geq \pi_B^u \quad \text{if and only if} \quad \rho_A \geq \frac{\sqrt{1 + r_A} + \sqrt{V r_A}}{V \sqrt{1 + r_A}}.
\]  

(12)

Figure 4 illustrates how lender \(A\)’s choice of interest rate, \(r_A\), and minimum credit rating, \(\rho_A\) affect lender \(B\)’s choice of whether to share the market or to undercut lender \(A\).

4.2 Stage I: Lender \(A\)

Lender \(A\) does not earn any profit if lender \(B\) undercuts in stage II. Figure 4 and equation (12) reveal that lender \(A\) can avoid being undercut if he sets \(\rho_A\) sufficiently high, relative to the interest rate \(r_A\), thereby specializing in giving loans to borrowers with sufficiently-high credit rating.
Therefore, lender $A$ can avoid being undercut by choosing any pair of $(r_A, \rho_A)$ satisfying (12) which is plotted on Figure 4. Thus, lender $A$ chooses the combination $(r_A, \rho_A)$ that solves

$$\max_{r_A, \rho_A} \pi_A = n(1 + r_A) \int_0^{\rho_A} \rho \, d\rho - n(1 - \rho_A) = \frac{n}{2} \left[ r_A(1 - \rho_A^2) - (1 - \rho_A)^2 \right],$$

subject to (12). Although we are not able to display an explicit solution for (13) we can still prove the following lemma.

**Lemma 1.** There exist a unique duopoly equilibrium. Formally, the profit maximization problem (13) has a unique solution.

**Proof.** From (13), lender $A$’s iso-profit curves are given by

$$r_A = \frac{2\pi_A + n(1 - \rho_A)^2}{n(1 - \rho_A^2)}. \quad (14)$$

Applying the Implicit-Function Theorem on (13), the slope of (14) is

$$\frac{dr_A}{d\rho_A} = \frac{1 - \rho_A^2}{2(r_A \rho_A + \rho_A - 1)} \geq 0 \quad \text{if and only if} \quad \rho_A \leq \frac{1}{1 + r_A}. \quad (15)$$

The inverse of the iso-profit (14) is plotted in Figure 5.

Figure 5 shows that lender $A$’s profit increases in the south-east direction (higher interest and...
higher market share). (14) implies that $r_A \to \infty$ as $\rho_A \to 1$. Equation (15) implies that the slope of these iso-profit curves may be negative for low values of $\rho_A$, but must turn positive for higher values. Hence, a unique solution of the maximization problem (16) exists where an iso-profit line of lender $A$ is tangent to (12).

4.3 Equilibrium with two lenders

Substituting $\rho_A$ from (12) into (13), lender $A$ solves

$$\max_{r_A} \pi_A = \frac{n(V - 1 - r_A) \left[ 2r_A \sqrt{V} - \sqrt{1 + r_A (V - 1 - V r_A)} \right]}{2V^2 \sqrt{1 + r_A}}.$$  \hfill (16)

We solve $A$’s problem by simulations. Our simulations proceed as follows. For each value of $V$, the “computer” determines the value of $r_A$ which maximizes (16). Then, substituting $r_A$ into (12) yields $\rho_A$. Substituting into (16) yields $\pi_A$. Finally, $r_B$, $\rho_B$, and $\pi_B$ are then determined by (9). Table 1 exhibits the equilibrium values for $V = 1.5, 1.8, 2$, and $3$.

Table 1 shows that both interest rates and profit levels rise with $V$, since a higher expected return on borrowers’ investments enables lending institutions to extract higher surpluses from the borrowers. However, with the improvement in financial conditions of the borrowers (higher return on their investment, captured by $V$) more of them become eligible for loans ($\rho_B$ decreases). Therefore, fewer borrowers are excluded from the loan market when competition among lenders
prevails. In addition, lenders’ profits increase with an increase in the borrowers’ population size \( n \).

Table 1 also reveals that bank \( B \), that lends to lower-quality borrowers, earns a higher profit compared with bank \( A \) that serves higher quality borrowers. Evidently, the interest charged by bank \( B \) more than compensates for the low quality of its customers. In addition, bank \( B \) maintains a significantly larger market share than bank \( A \). Table 1 reveals the following result.

**Result 3.** In a subgame-perfect equilibrium, an increase in borrowers’ return on their investments increases the market size of both lending institutions as both lenders lower their minimum credit rating for loan eligibility. Formally, both \( 1 - \rho_A \) and \( \rho_A - \rho_B \) increase as \( V \) increases.

The increase in the degree of competition in the loan market is beneficial to borrowers. This is illustrated in Figure 6. Figure 6 illustrates how the surplus is divided between borrowers and the two lending institutions. The maximum extractable total surplus of a set of consumers is the area under the steeper (upper) line. The upper line reveals that lender \( B \) extracts the entire surplus from borrowers indexed on \([\rho_A, \rho_B]\). In contrast, the lower line demonstrates that lender \( A \) shares the surplus with its borrowers who are indexed on \([\rho_A, 1]\) as it cuts through the area below the upper line (hence surplus is divided between lender \( A \) and the borrowers). Comparing Figure 6 with Figure 2 reveals that the entry of lender \( B \) into the market is beneficial to borrowers with high credit ratings, who, after entry occurs, gain strictly positive surplus from taking loans.

<table>
<thead>
<tr>
<th>( V )</th>
<th>Bank ( A )</th>
<th>Bank ( B )</th>
<th>Market shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_A )</td>
<td>( \rho_A )</td>
<td>( \pi_A )</td>
<td>( r_B )</td>
</tr>
<tr>
<td>1.5</td>
<td>0.312</td>
<td>0.889</td>
<td>0.026n</td>
</tr>
<tr>
<td>1.8</td>
<td>0.485</td>
<td>0.852</td>
<td>0.055n</td>
</tr>
<tr>
<td>2</td>
<td>0.603</td>
<td>0.835</td>
<td>0.077n</td>
</tr>
<tr>
<td>3</td>
<td>1.135</td>
<td>0.782</td>
<td>0.196n</td>
</tr>
</tbody>
</table>

**Table 1:** Duopoly equilibrium values under sequential entry.
5 Regulated Interest Rate

Suppose now that the regulating authority imposes a uniform interest rate in the market for loans, which we denote by \( \bar{r} \). However, suppose that each lending institution can freely set its minimum credit rating which makes borrowers eligible for loans. Our analysis maintains the same order of moves (stage I, and II) as in previous sections. That is, lender \( A \) sets \( \rho_A \) in stage I, and lender \( B \) sets \( \rho_B \) in stage II. Note that the regulator must set \( \bar{r} < V - 1 \), as otherwise the utility function (2) implies that no borrower would find it beneficial to borrow and invest.

5.1 Market consequences

The profit of a loan-making institution, from making a loan to a borrower with credit rating of \( \rho \), is \( \rho(1 + \bar{r}) - 1 \). This is the expected payment of the principal plus the interest rate minus the $1 loaned to the borrower. Therefore,

\[ \rho V - 1 = (1 + \rho_B)\rho - 1 \]

\[ (1 + \rho_A)\rho - 1 \]

Figure 6: Division of surplus between borrowers and lending institutions under competition.

---

6 Interest determination is only one form of bank regulation that could be considered. Other common measures include capital adequacy requirements, restrictions on banks’ ownership of non-financial firms, reserved requirements, implicit or explicit deposit insurance and various restrictions on asset and liability structures. See Demirgüç-Kunt and Kane (2002) and Sironi (2002) for discussions of regulatory measures in the banking sector that can affect competitive outcomes.
Result 4. When a uniform interest rate is mandated, no institution would lend to customers whose credit rating is below $\hat{\rho} = 1/(1 + \bar{r})$.

Proof. The profit made on borrower $\rho$ is nonnegative only if $\rho(1+r) - 1 \geq 0$, hence if $\rho \geq 1/(1+\bar{r})$. 

Therefore, the regulator can maintain an indirect control over which type of borrowers are excluded from the loan market by varying the mandated interest rate. By setting a lower $\bar{r}$, the regulator increases the number of borrowers who are ineligible to obtain loans from any institution. In contrast, by setting the highest-possible interest rate, $\bar{r} = V - 1$, the regulator lowers the exclusion level to $\tilde{\rho} = 1/V$ which is the same level as maintained under monopoly given in equation (5).

5.2 Equilibrium under a regulated interest rate

Given a mandated uniform interest rate, in stage I lender $A$ sets its minimum credit rating, $\rho_A$, and in stage II lender $B$ sets $\rho_B$. Since both lending institutions now charge the same interest rate, we wish to specify how borrowers are allocated between the lending institutions in the case that they are eligible to borrow from both institutions.

Assumption 1. If the $n$ borrowers with a credit rating of $\rho$ are eligible to borrow from both institutions and if both institutions charge the same interest rate, the $n$ borrowers are equally divided between the lending institutions.

Here we adopt a simple formulation that captures the symmetry between an incumbent lender and a new entrant. Note that Assumption 1 was not needed in Section 4 where lenders could freely set both interest rates and minimum credit ratings, since in Section 4 lenders end up setting different interest rates.

In the second stage, given $\rho_A \geq 1/(1 + \bar{r})$ (see Result 4), lender $B$ sets a low credit requirement given in Result 4, thereby monopolizing over borrowers indexed on $[1/(1 + \bar{r}), \rho_A]$ and sharing with lender $A$ borrowers indexed on $[\rho_A, 1]$. Thus, by Assumption 1, since lender $A$ equally shares the market with lender $B$, lender $A$ maximizes its profit (market size) by also setting $\rho_A = 1/(1+\bar{r})$. Therefore,
**Result 5.** When lending institutions are forced to charge a regulated uniform interest rate, they will all set the same minimum credit rating required from borrowers. Formally, the unique equilibrium minimum credit ratings are $\rho_A = \rho_B = 1/(1 + \bar{r})$.

Comparing Table 1 with Result 5 reveals that the mandated interest rate policy eliminates the incentives of lenders to differentiate themselves by lending to different groups of borrowers.

### 5.3 Welfare consequences

The market for loans analyzed in this paper has a continuum of heterogeneous potential borrowers and two loan-making institutions. Looking first at how the imposition of a uniform interest rate affects borrowers, we can see that such a policy affects different borrowers in different ways. First, there are borrowers who gain since the mandated interest rate is lower than the market-determined rate (borrowers who pay an interest $r = V - 1$ under market competition, but a lower rate when the rate is regulated). In contrast, Result 4 implies that a lower mandated interest rates increases the number of borrowers who are excluded by lending institutions. Therefore, borrowers who become excluded as a result of the imposition of a lower interest rate are worse off as a result of this policy. Second, it may happen that for some borrowers, in particular, borrowers with high credit rating who take loans from lender $A$ (at a lower interest rate) also become worse off with the imposition of a uniform interest rate. Thus,

**Result 6.** The imposition of a mandated uniform interest rate is Pareto noncomparable to a market equilibrium.

Next, we define the economy’s social-welfare function as the sum of nonexcluded borrowers’ utilities and the profit of the two loan-making institutions. Formally,
\[
W \overset{\text{def}}{=} \int_{\rho}^{1} U_{\rho} d\rho + \pi_{A} + \pi_{B}
\]

\[
= n \int_{\rho_{B}}^{\rho_{A}} \rho [V - (1 + r_{B})] d\rho + \frac{n}{2} \int_{\rho_{A}}^{1} \rho [V - (1 + r_{B})] d\rho + \frac{n}{2} \int_{\rho_{A}}^{1} \rho [V - (1 + r_{A})] d\rho
\]

\[
+ n \int_{\rho_{B}}^{\rho_{A}} \rho (1 + r_{B}) - 1 \ d\rho + \frac{n}{2} \int_{\rho_{A}}^{1} [\rho (1 + r_{B}) - 1] \ d\rho + \frac{n}{2} \int_{\rho_{A}}^{1} [\rho (1 + r_{A}) - 1] \ d\rho
\]

\[
= n \int_{\rho_{B}}^{1} (\rho V - 1) \ d\rho.
\]

Thus, social welfare is measured by the real investment effect which is the sum of the expected “revenue” \((\rho V)\) minus the real cost which is $1 for each borrower.\(^7\) This outcome should come as no surprise since interest payments constitutes only a transfer from borrowers to lending institutions and therefore must cancel out in the social welfare function. Therefore,

**Result 7.** Social welfare is reduced when the regulator mandates a uniform interest rate on loans. Further, social welfare monotonically declines with a decrease in the mandated market uniform interest rate.

**Proof.** Maximizing the social welfare function (17) implies that the socially-optimal minimum credit rating eligible for a loan is \(\rho^{*} = 1/V\). However, Result 4 implies that the equilibrium minimum credit rating is \(\bar{\rho} = 1/(1 + r) > \rho^{*}\) for mandated interest \(r < V - 1\). Further, \(\bar{\rho} - \rho^{*} = 1/V\) increases when \(r\) declines, thus, the number of excluded borrowers beyond the socially-optimal level increases with a decline in \(r\).

The intuition behind Result 7 is as follows. Interest-rate regulation by itself does not affect aggregate social welfare. But, this regulation does have an indirect effect on social welfare by increasing the number of excluded potential borrowers.\(^7\)

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\(^7\)The same idea is expressed in (McGrattan and Prescott, 2006, Appendix A pp.24–28).
6 Alternative Market Structures

So far, our analysis was conducted under a sequential-entry market structure where lender $A$ had to set its minimum credit requirement and its interest rate before the entry of lender $B$. A natural question to ask at this point is how robust our results are to changes in the market structure in this market for loans.

In order to investigate this question, in this section we look at an alternative market structure assuming that both lenders coexist from the start, and make decisions in two stages.

**Stage I:** Each lender $i$ takes the minimum credit rating of lender $j$, $\rho_j$, as given and chooses its own minimum credit rating $\rho_i$, where $i, j = A, B$ and $i \neq j$.

**Stage II:** Both lenders take $\rho_A$ and $\rho_B$ as given. Then, each lender $i$ takes the interest rate of lender $j$, $r_j$, as given and chooses its own lending rate $\rho_i$.

Ideally, we look for a subgame perfect equilibrium for this simultaneous-move two stage game. However, as we now demonstrate such an equilibrium may not exist unless we slightly modify this equilibrium concept.

6.1 Equilibrium interest rates in the second stage

Let $\rho_A$ and $\rho_B$ be given, and suppose with no loss of generality that $\rho_B \leq \rho_A < 1$, as illustrated in Figure 3. By a way of contradiction, suppose that $(r_A, r_B)$ is a Nash equilibrium (NE in what follows) for the second stage interest rate competition.

Given $r_B$, lender $A$ maximizes profit by raising its interest rate to $r_A = r_B - \epsilon$ without losing any of its $1 - \rho_A$ borrowers, where $\epsilon$ is a small number. However, given $r_A \approx r_B$, lender $B$ can raise its interest to $V - 1$ (monopoly rate) without losing its monopoly over its $\rho_A - \rho_B$ borrowers. Altogether we have $r_A \approx r_B = V - 1$. However, Figure 4 implies that these interest rates fall in the range where lender $B$ can raise its profit by undercutting $A$ by setting $r_B = r_A - \epsilon$, which completes a full circle. Hence, $(r_A, r_B)$ is not a NE.

The nonexistence of a Nash equilibrium is not unique to the present setup and occurs in other “location” type models as reported in d’Aspremont, Gabszewicz, and Thisse (1979). Instead of
resorting to a mixed strategy solution, we utilize a simple equilibrium concept in pure actions.

Intuitively, the pair of interest rates \((\bar{r}_A, \bar{r}_B)\) is an *Undercut-proof Equilibrium* (UPE in what follows) if each lender maximizes its interest rate subject to not being undercut by the rival lender. Undercutting occurs when the rival lender sets a lower rate thereby grabbing the entire market, see formal definitions in Shy (2001, 2002). In the present case, since \(\rho_B < \rho_A\), lender \(B\) is the sole lender to borrowers indexed on \([\rho_B, \rho_A]\) and hence cannot be undercut by lender \(A\). Therefore, as before, (2) implies that \(r_B\) is maximized at \(r_B = V - 1\).

In an UPE, lender \(A\) maximizes \(r_A\) subject not to be undercut by lender \(B\). Formally, lender \(A\) raises \(r_A\) maintaining that (8) is no less than (10), formally

\[
\frac{\partial \pi_A}{\partial \rho_A} = \frac{n[V^3 \rho_A (2\rho_A^2 - 1)V^2 \rho_A (2 - 3\rho_A) - V \rho_A + 1]}{1 - V^2} = 0 \tag{19}
\]

\[
\frac{\partial^2 \pi_A}{\partial \rho_A^2} = \frac{V n[V^2 (6\rho_A^2 - 1) + 2V (1 - 3\rho_A) - 1]}{1 - V^2}.
\]

Parallel to Table 1, Table 2 exhibits the equilibrium values for \(V = 1.5, 1.8, 2, \) and 3. Again, higher investment returns, \(V\), reduce the size of the excluded group. The values of \(\rho_A\) and \(r_A\) are obtained from (19) and then from (18). Table 3 compares Table 2 with Table 1 and lists the differences between simultaneous and sequential moves lending industries. Table 3 implies the following list of results.

**Result 8.** (a) Lender \(A\) earns a higher profit, maintains a larger market share, and pays a lower interest
Table 2: Equilibrium under simultaneous moves

<table>
<thead>
<tr>
<th>( V )</th>
<th>Bank A</th>
<th>Bank B</th>
<th>Market shares</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( r_A )</td>
<td>( \rho_A )</td>
<td>( \pi_A )</td>
</tr>
<tr>
<td>1.5</td>
<td>0.308</td>
<td>0.867</td>
<td>0.029n</td>
</tr>
<tr>
<td>1.8</td>
<td>0.478</td>
<td>0.827</td>
<td>0.060n</td>
</tr>
<tr>
<td>2</td>
<td>0.588</td>
<td>0.809</td>
<td>0.083n</td>
</tr>
<tr>
<td>3</td>
<td>1.115</td>
<td>0.760</td>
<td>0.207n</td>
</tr>
</tbody>
</table>

Table 3: Simultaneous versus sequential moves

<table>
<thead>
<tr>
<th>Variable</th>
<th>Notation</th>
<th>Simultaneous Moves</th>
<th>Sequential Moves</th>
</tr>
</thead>
<tbody>
<tr>
<td>A’s interest</td>
<td>( r_A )</td>
<td>lower</td>
<td>higher</td>
</tr>
<tr>
<td>B’s interest</td>
<td>( r_B )</td>
<td>same</td>
<td>same</td>
</tr>
<tr>
<td>A’s market</td>
<td>( 1 - \rho_A )</td>
<td>larger</td>
<td>smaller</td>
</tr>
<tr>
<td>B’s market</td>
<td>( \rho_A - \rho_B )</td>
<td>smaller</td>
<td>larger</td>
</tr>
<tr>
<td>A’s profit</td>
<td>( \pi_A )</td>
<td>higher</td>
<td>lower</td>
</tr>
<tr>
<td>B’s profit</td>
<td>( \pi_B )</td>
<td>lower</td>
<td>higher</td>
</tr>
</tbody>
</table>

under the simultaneous move game compared with being the first mover in a sequential game.

(b) Lender B earns a lower profit and maintains a smaller market share under the simultaneous move game compared with being the last mover in a sequential game.

Result 8 points out a “first-mover disadvantage” in the market for loans. More precisely, in the sequential-moves game lender A maintains a higher credit requirement (thus, a lower market share) to prevent lender B from undercutting during the second stage. This effect is mitigated when both lenders move simultaneously.

7 Summary and Discussion

Most loan markets are characterized by borrowers who have to meet some basic quality requirements. This is often the situation in small business markets or in consumer’s loan markets, where the lending bank determines not only the interest rate on loans, but also some minimum credit rating that a potential borrower should have in order to be eligible for a loan. In this paper we analyzed an imperfectly-competitive market for loans, where the banks set both the interest rate and minimum credit rating that potential borrowers should possess. We investigated the impact of
competition on loan market shares in such an environment and the degree of exclusion. We also analyzed the effect of interest regulation on social welfare.

We used a model in which banks choose how much to lend given the quality of loans. In this setting we modeled specifically the nature of competition between two competing banks. The results helped to identify the quality-cost trade off and its impact on market share. We also showed how the competitive results are influenced by regulation.

Specifically, we found that a single lender, in the loan market, will practice some degree of credit rationing. That is, borrowers whose expected net yield on investment is negative, will be excluded. When a second lender competes in the loan market it may not use outright price competition in order to gain market share. Instead, it is likely to charge a higher interest rate and serve borrowers with a lower credit rating than the incumbent bank. In a sub-game perfect equilibrium, both lending firms increase their market share by lowering their minimum credit rating. Table 4 summarizes our results how social surplus is divided between lenders and borrowers under the analyzed market structures.

<table>
<thead>
<tr>
<th>Project Value V</th>
<th>Social Surplus SW</th>
<th>Monopoly π^m CS^m</th>
<th>Duopoly π_A π_B CS^d</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.083</td>
<td>0.083</td>
<td>0.026 0.037 0.020</td>
</tr>
<tr>
<td>1.8</td>
<td>0.177</td>
<td>0.177</td>
<td>0.060 0.066 0.050</td>
</tr>
<tr>
<td>2</td>
<td>0.250</td>
<td>0.250</td>
<td>0.077 0.112 0.061</td>
</tr>
<tr>
<td>3</td>
<td>0.667</td>
<td>0.667</td>
<td>0.196 0.302 0.169</td>
</tr>
</tbody>
</table>

Table 4: Division of social surplus among lenders and borrowers.
Note: All entries should be multiplied by \( n \).

We also consider a situation where a financial sector regulator imposes a uniform mandated interest rate in the loan market. We find that in such a situation credit rationing will still prevail and a large number of potential borrowers will be excluded. The regulator, however, may reduce the amount of rationing by raising the mandated interest rate. Under interest rate regulation, all borrowers are equally divided between the lending institutions. Since both charge the same regulated rate, they also set the same minimum credit rating requirements. A regulated interest rate essentially eliminates the incentives of lenders to compete by lending to different group of borrowers. Social welfare, however, is reduced when a uniform interest rate is imposed.
References


