Measuring Uncertainty in Monetary Policy Using Implied Volatility and Realized Volatility*

Bo Young Chang
Bank of Canada

Bruno Feunou
Bank of Canada

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Abstract

We measure uncertainty surrounding the central bank's future target interest rates using implied volatility (IV) computed from interest rate option prices and realized volatility (RV) computed from intraday prices of interest rate futures. Both measures are available daily and reflect the expectation based on asset prices, providing useful addition to survey-based measures. Based on our two volatility measures, we find that uncertainty decreased on average following the Bank of Canada's target rate announcements. We also find that some of the most important policy actions taken by the Bank of Canada as a response to the financial crisis of 2007-2008, including the conditional commitment of 2009-2010, unscheduled cut in the target rate coordinated with other major central banks, and introduction of term purchase and resale agreements (PRAs), all reduced uncertainty.

JEL Classification: E4

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1 Introduction

One of the main tasks of a central bank is to manage the market's expectation of future monetary policy decisions. In particular, central banks spend a large amount of resources in formulating the path of their target interest rate so as to minimize uncertainty in short term interest rates. To assess the effectiveness of their strategies, central banks then need appropriate measures of ex-ante uncertainty.

We propose two measures of uncertainty around future target rates based on prices of interest rate derivatives. The first measure is implied volatility computed from prices of options on interest rate futures. The second measure is realized volatility computed from intraday prices of interest rate futures. In the case of Canada, which is the focus of this study, we identify BAX contracts, which are futures on the 3-month CDOR (average bankers' acceptances rate), and OBX contracts, which are options on BAX, as two interest rate derivatives best suited for our purpose.

We check that BAX IV and BAX RV capture the amount of uncertainty around future target rates by looking at their relation to future realized uncertainty. Realized uncertainty is measured by the magnitude of the difference between ex-ante expectation of future overnight rates (rate targeted by the central bank in Canada) implied from overnight index swap (OIS) rates and actual realized overnight rates. We find that both BAX IV and RV are positively related to future realized uncertainty as expected.

BAX IV and RV are then used to assess impact of different policy announcements made by the Bank of Canada (BoC). We find that uncertainty decreased on average following target rate announcements. We also find that some of the most important policy actions taken by the BoC as a response to the financial crisis of 2007-2008, including the conditional commitment of 2009-2010, unscheduled cut in the target rate coordinated with other major central banks, and introduction of term purchase and resale agreements (PRAs), all reduced uncertainty.

We also explore the relation between our two measures of uncertainty and the risk premium in the OIS market, based on the intuition that uncertainty, or risk, is often closely related to risk premium in the prices of related derivative securities. We show that BAX IV and RV are positively related to risk premium in OIS rates, thus can be used to adjust OIS-implied expectations of future target rates for a risk premium. We find that the resulting risk-adjusted OIS rates are closer to realized overnight rates than unadjusted OIS rates, providing improvement in prediction of future target rates.

An alternative measure of uncertainty often used by the central banks is the dispersion of opinions in a survey of target rate forecasts, but this measure suffers from several important limitations. First, surveys are conducted infrequently. The most popular ones, which are published by Bloomberg and Reuters, are conducted shortly before each pre-scheduled target rate announcements in both the US and Canada. As a result, surveys cannot be used to assess impact of other macroeconomic events or central bank announcements occurring outside of target rate announcement dates. Second, surveys are based on a limited number of market participants, so might not accurately reflect the expectation of all the participants in the market. Third, survey respondents are professional market participants, thus their forecasts are potentially affected by their career concerns. Since BAX IV and RV are available daily and reflect the expectation based on asset prices, our volatility measures do not suffer from the limitations of survey-based measures.

Literature review

Most of the related literature focuses on changes in the direction of the future path of monetary policy. See, for example, the survey in Gürkaynak, Sack, and Swanson (2006). A few studies examine changes in uncertainty around future target rates. Amin and Ng (1997) test whether implied volatility of eurodollar futures options can predict future realized volatility of the LIBOR rate. Neely (2005) also examines the ability of implied volatility of eurodollar futures options to predict realized volatility of eurodollar futures, and link changes in implied volatility to macroeconomic events. Swanson (2006) shows that the financial markets' uncertainty about the eurodollar rate, measured by change in the level of implied volatility of eurodollar options, trended downward very strongly since 1989. Our study adds to the existing literature by looking at both implied volatility and realized volatility, and by applying them to the Canadian data.

Our study is closest to Bauer (2012) which examines the evolution of interest rate uncertainty between 2003 and 2012 using implied volatility of eurodollar options. They find that the Quantitative Easing (QE) program announcements and forward-looking statements in the FOMC are followed by drop in implied volatility, indicating that these policy actions succeeded in lowering uncertainty around future interest rates. Among the statements made by the Federal Reserve Board announcing various QEs and forward guidance during this period, the largest drop in implied volatility (13 bps over one year horizon) is found to have occurred after the announcement of the conditional commitment in August 2011. Unlike in the US where the central bank's conditional commitment is yet to be removed, the BoC has ended its commitment in April 2010. Thus, our study provides valuable insights into possible impact of the Federal Reserve's eventual exit from its conditional commitment.

Carson, Craig, and Melick (2005) and Emmons, Lakdawala, and Neely (2006) go beyond implied volatility and extract option-implied probability distribution of future interest rate expectation using federal funds futures options. We cannot conduct the same exercise due to a lack of a Canadian instrument that is equivalent to the federal funds futures options. We only have options on BAX whose underlying interest rate is the 3-month CDOR, not the central bank's target rate.

Relevant studies in Canada include Johnson (2003) and Fay and Gravelle (2010). Johnson (2003) compares different measures of target rate expectations in Canada using treasury bills, bankers' acceptance (BA) rates, FX forwards, and BAX futures. He recommends using BA rates for one to three month horizons and BAX prices for longer than three month horizons. Fay and Gravelle (2010) test whether inclusion of forward-looking statements in monetary policy communications made the BoC more transparent by using absolute changes in BAX as a measure of surprise. We also use BAX futures and OBX options (options on BAX) in our study, consistent with Johnson (2003) and Fay and Gravelle (2010).

The rest of the paper is organized as follows. Section 2 presents the theory, data, and empirical methodologies behind the computation of volatility measures. Section 3 presents empirical results on the impact of BoC announcements on uncertainty. Section 4 explores relation between our uncertainty measures and OIS excess returns. Section 5 concludes.

2 Constructing measures of uncertainty

We propose two volatility measures computed from interest rate derivatives as measures of uncertainty around future BoC target rates,

2.1 Theory of implied volatility and realized volatility

Consider a futures contract that references a 3-month interest rate, r_T, at the futures expiry, T. Let f_t be the price of this futures at time t ($t \leq T$). Then we have,

$$f_t(T) = E_t^Q[r_T].$$

Hence, the expected level of r_T can be implied from the price of the futures contract that expires at time T, $f_t(T)$.

Similarly, the amount of uncertainty about r_T at time t can be captured by the expected variance of r_T at time t,

$$VAR_t(T) = E_t^Q\left[(r_T - \overline{r_T})^2\right]. \tag{1}$$

In practice, variance under a risk-neutral measure, Q, cannot be obtained from futures price, but requires options on the underlying interest rate, r_T, or options on the futures on r_T.

Suppose r_t follows a stochastic process of the form,

$$dr_t = \alpha_t dt + \sigma_t dW_t + \sum \Delta r_s,$$

where $\Delta r_t \equiv r_t - r_{t-}$, denotes a jump in r at time t. The total variation of r between $[t, T]$ is defined through the quadratic variation,

$$[r, r]_T - [r, r]_t \equiv \int_t^T \sigma_s^2 ds + \sum |\Delta r_s|^2 .$$

The quadratic variation is closely related to the expected variance defined in (1), and is measured through the realized variance defined as follows. For an increasing sequence of random partitions of $[t, T]$, $t = t_0 \leq t_1 \leq \cdots \leq t_N = T$,

$$RV_{[t,T]} \equiv \sum_{j=1}^{N} \left(r_{t_j} - r_{t_{j-1}}\right)^2$$

Barndorff-Nielsen and Shephard (2002) shows that if sampled frequently, the realized variance converges in probability to the quadratic variation,

$$\sup_{j\geq 1} (t_j - t_{j-1}) \to 0 \Longrightarrow RV_{[t,T]} \to [r, r]_T - [r, r]_t .$$

Finally, the option implied variance is by definition the expected quadratic variation,

$$IV_t(T) = \frac{1}{T-t} E_t^Q \left[\int_t^T \sigma_s^2 ds + \sum |\Delta r_s|^2\right] \approx \frac{1}{T-t} E_t^Q \left[RV_{[t,T]}\right] .$$

2.2 Data

In Canada, the overnight rate at which major financial institutions borrow and lend one-day funds among themselves is the main tool used by the BoC to conduct monetary policy. The *Canadian Overnight Repo Rate Average (CORRA)* is the weighted average of overnight rates, and is the rate targeted by the central bank. In terms of derivatives on the CORRA, there are two futures products traded on the Montreal Exchange (ONX and OIS futures) and Overnight Indexed Swaps (OIS) traded over the counter. Options on CORRA are yet to be introduced. The futures, ONX and OIS futures, were introduced in 2002 and 2012 respectively, but their liquidity is still quite limited. As a result, OIS contracts are the main hedging instruments for the overnight rate in Canada, making OIS rates the best gauge of the market's expectation of future overnight rates.

Another important benchmark for short term interest rate in Canada is the *Canadian Dealer Offered Rate (CDOR)*, which is the average of Canadian bankers' acceptance rates for specific terms-to-maturity. CDOR is comparable to the LIBOR in the US. Similarly to the US, there is a liquid market on futures on the 3-month CDOR and options on these futures, called BAX futures and OBX options respectively.

Although ideal instruments for our study would be the futures and options on CORRA itself, we use BAX futures and OBX options instead due to limited liquidity of futures and absence of options on CORRA as mentioned earlier. The use of BAX and OBX is consistent with the use of Eurodollar futures and options in many related studies in the literature.

We obtain daily time series of CORRA and OIS rates from Bloomberg, and the following data on BAX and OBX from the Montreal Exchange:

- BAX Intraday Quotes (January 2002 – May 2011)
- BAX Trades (January 1997 – March 2013)
- OBX End-of-day (February 2005 – March 2013)
- OBX Trades (January 2005 – June 2011).

<Table 1> compares liquidity of BAX and OBX contracts in terms of average daily trading volume and average daily number of trades. The trading volume of BAX contracts are roughly 20 times that of OBX contracts. In number of trades, the difference is an order of magnitude larger, around 400 times larger for BAX compared to OBX. This is due to the fact that the average size of OBX trades is much larger than that of BAX trades. In terms of open interest, as of late March 2013, the size of the market for BAX is approximately CDN$600 billion compared to CDN$70 billion for OBX in notional amount. Thus, a considerable amount of money is at stake in the market for both BAX and OBX.

< Table 1: Liquidity of BAX and OBX>

Note that there was zero trading of OBX in 2009. This temporary stop in trading began in October 2008 at the onset of the subprime crisis in the US, and continued until the end of the BoC's conditional commitment, which lasted between April 2009 and April 2010. Trading in OBX resumed in mid-March 2010, about one month before the removal of the commitment was announced to the public.

We also report in <Table 2> the average daily number of OBX contracts with positive open interest. We only consider options with positive open interest so as to filter out any option with stale price that is not informative of the market's current assessment of the future. The average daily total number of options is around 20. When grouped by maturity, we observe that most of the options fall into the maturities between one month and six months.

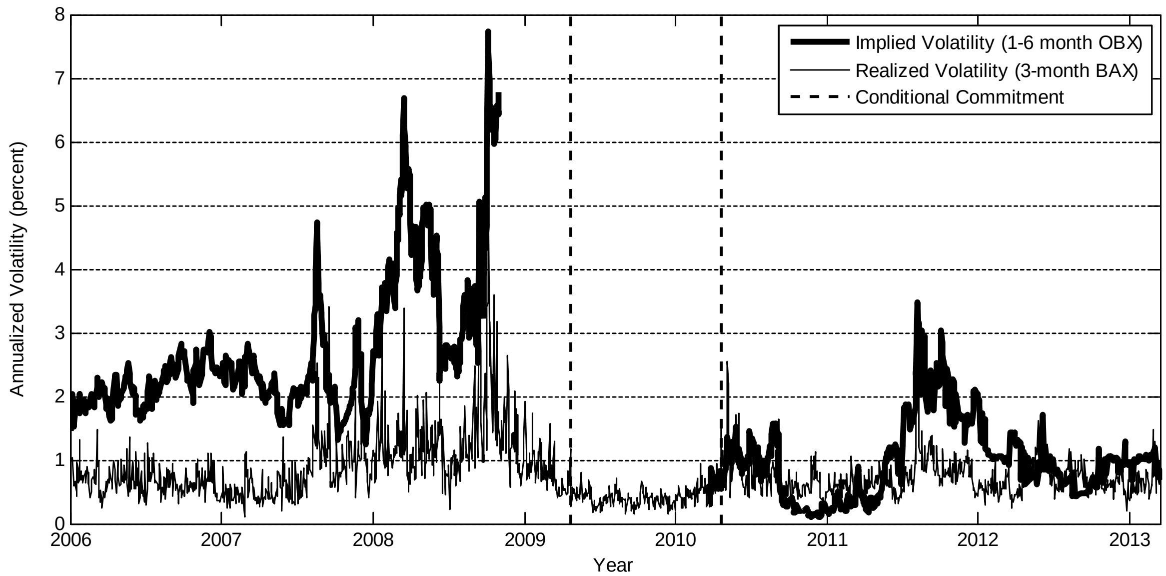
< Table 2: Average daily number of OBX options by maturity>

2.3 Implied volatility of BAX

We compute implied volatility of BAX from OBX option prices using an option valuation formula based on the Vasicek model (1977) of short rate. Most of the past studies looking at information content of implied volatility of the short term interest rate use the Barone-Adesi and Whaley (1987) approximation of the American option pricing model to compute implied volatility. The option pricing formula based on the Vasicek model provides an improved implied volatility estimation by taking into account the stochastic characteristics of the interest rate process. Amin and Ng (1997) study the ability of implied volatility of eurodollar futures options in forecasting the future volatility of the eurodollar futures rate. They compare implied volatilities computed based on five different volatility models: (1) Ho-Lee (1986), (2) CIR (1985), (3) Courtadon (1982), (4) Vasicek (1977), and (5) Linear Proportional (Heath-Jarrow-Morton), and find that the Vasicek (1977) and linear proportional volatility models perform better than the other implied volatility models. Details of the methodology and implementation are provided in Appendix A.

Ideally, we would like to compute implied volatilities for different maturities so that we can examine the term structure of implied volatility. However, due to relatively low liquidity of OBX, we cannot compute implied volatility of BAX for different maturities in a consistent manner over time, so we compute one implied volatility for each day using all options of maturities from one to six months.

The daily times series of BAX IV is plotted in <Figure 1>. BAX IV reached its highest level of around 8% in late 2008 during the US subprime crisis, and its lowest level of around 0.25% in 2010-2011 when the target rate was kept extremely low following the crisis. Compared to the VIX, which ranged from approximately 10% to 80% in the same period, implied volatility of BAX is significantly smaller in magnitude. This is consistent with the fact that uncertainty in the short term interest rate is much lower than uncertainty in the stock market.



<Figure 1> Implied volatility (IV) and realized volatility (RV) of BAX

Implied volatility could not be computed between November 2008 and mid-March 2010 due to a lack of trading in OBX contracts during this period. This temporary stop in trading began in October 2008 at the onset of the subprime crisis in the US, possibly due to prohibitively high margin requirements caused by high volatility of the underlying interest rate and high volatility risk premium at the time.

This period also coincides with the duration of the BoC conditional commitment between April 2009 and April 2010. On April 21, 2009, the BoC announced that, "Conditional on the outlook for inflation, the target overnight rate can be expected to remain at its current level until the end of the second quarter of 2010 in order to achieve the inflation target." The commitment was eventually removed on April 20, 2010, and the target rate was raised to 0.50% at the following FAD on June 1, one month earlier than was promised in the initial commitment. As expected, the commitment led to a decline in the amount of uncertainty in the target rate expectation during the commitment period as indicated by the low level of BAX RV shown in <Figure 1>.

Although we cannot compute BAX IV throughout the conditional commitment period, we observe that the level of IV at the time of trading resumption is significantly lower than it was when the trading halted in late 2008, consistent with the evidence of decreased uncertainty exhibited by the low level of RV throughout the conditional commitment period.

The timing of trading resumption in OBX contracts also provides an interesting insight into the market's expectation on the timing of the removal of the conditional commitment.

<Figure 1> shows that trading of OBX contracts (June 14, 2010 expiry) resumed in mid-March of 2010, one month before the removal was announced to the public. This trading pattern suggests that the market anticipated a possible early removal of the commitment before the actual announcement was made to this effect.

The BoC's removal of the commitment resulted in a large increase in the level of both IV and RV compared to that observed during the conditional commitment period. However, both IV and RV are much lower than they were during the crisis between late 2007 and late 2008, and RV is comparable to its level just before the beginning of the commitment.

The Canadian experience of exit from a conditional commitment described above can shed some light into what we can expect to see following the Federal Reserve Board's eventual exit from its conditional commitment. In the US, the Federal Reserve Board (the Fed, henceforth) announced its own conditional commitment in August 2011 when the FOMC noted that economic conditions "are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013." Since the initial announcement, the end of the commitment of mid-2013 has been extended first to late 2014 in January 2012, then again to mid-2015 in September 2012. It is difficult to make a direct comparison of different conditional commitments since impact of each commitment will depend on factors such as credibility of the commitment and other exogenous factors. For example, the end date of the commitment by the Fed has been extended twice so far, which could have affected the market's confidence in the end date of the commitment. Nevertheless, if in fact the conditional commitment by the Fed had the effect of reducing uncertainty around future policy rates substantially, similarly to the experience in Canada, then we would expect to see a similar increase in uncertainty following the Fed's future exit from its conditional commitment.

The BoC's conditional commitment is one example of the central bank's policy actions for which BAX IV and RV served as effective gauges of uncertainty around future target rates. We present the results of other applications of BAX IV and RV in Section 3 of this paper.

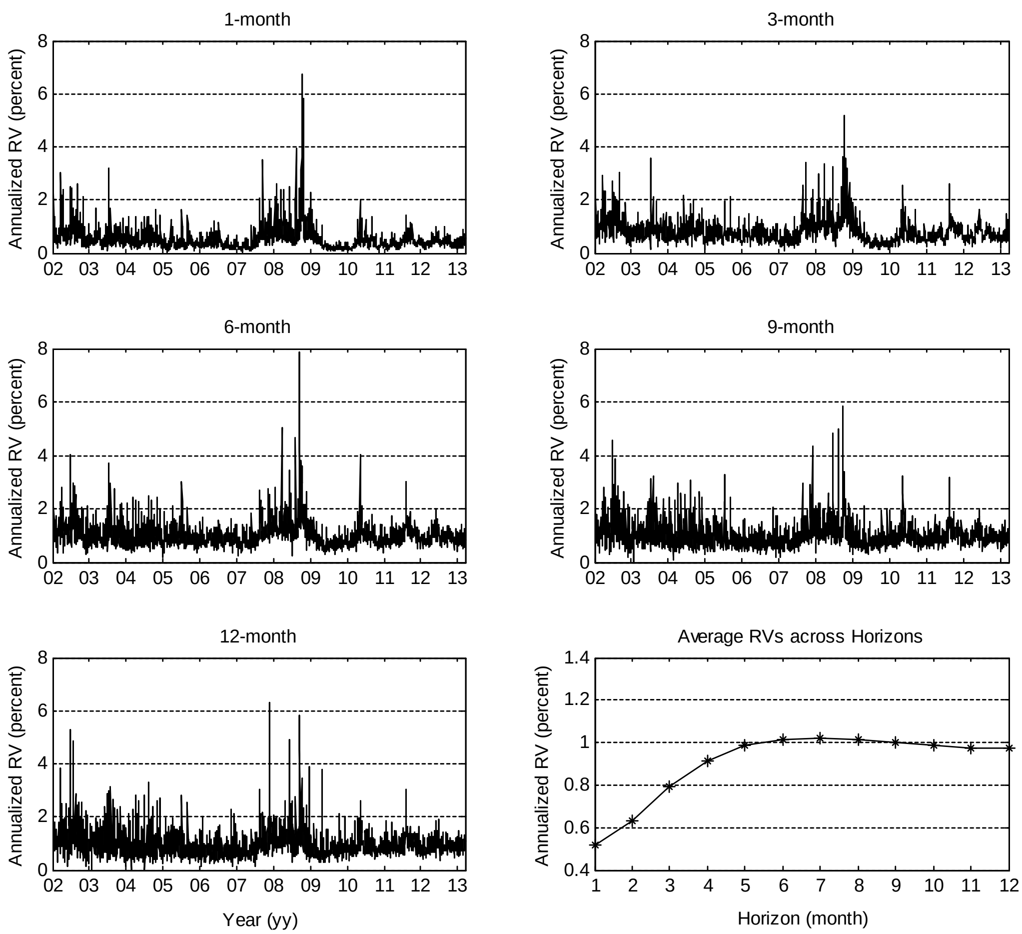
2.4 Realized volatility of BAX

Realized volatility computed from high-frequency intraday prices is another way to measure uncertainty in the market's expectation of the future interest rate. Following Andersen and Bollerslev (1998), Barndorff-Nielsen and Shephard (2002), and Meddahi (2002), we compute realized volatility as,

$$RV = \sqrt{\sum_{i=1}^{N-1} (S_{i+1} - S_i)^2 + \Delta S^2_{overnight}},$$

where $\Delta S_{overnight}$ is the change in the BAX price between the close of the previous day and the opening of the current day, and S_i are the intraday prices observed at a certain interval (e.g. every 5 min. or every 5 tick). In our implementation, we sample intraday trade prices of BAX at the interval of 5 trade ticks. Details of the methodology and implementation are provided in Appendix B.

The average level of BAX RVs with different maturities are plotted in <Figure 2>. The plot in the bottom right panel shows that the interest rate uncertainty captured by BAX RV peaks at around 6-month maturity, then flattens after that. The substantially lower level of RV in the short maturities indicates that there isn't much disagreement on the future path of the monetary policy in very short horizons of one to two months.



<Figure 2> Realized volatility of BAX

<Figure 1> plots the times series of 3-month maturity RV together with IV. We choose

the 3-month maturity for RV because the average maturity of options used in the computation of IV is around 3 month. Similar to IV, RV peaks around October 2008, and reaches its lowest level during the BoC's conditional commitment period.

We find that IV was higher than RV in the earlier period of 2006-2008 with a dramatic increase in the IV-RV spread in late 2008. In the later period between 2010-2013, however, IV is often lower than RV. The difference between IV and RV is known as the volatility risk premium (or variance risk premium if based on squared volatility) in the literature. Our finding that on average IV is higher than RV for BAX is consistent with existing findings in the stock market. Although we do not investigate what is driving the volatility risk premium of BAX in the present study, the volatility risk premium of BAX contains potentially useful information about risk and/or attitude towards risk of investors in the market for short term interest rate in Canada. For instance, Bollerslev, Tauchen, Zhou (2009) find that the volatility risk premium of the S&P 500 index is a good predictor of the future stock market return. We leave a detailed investigation of the volatility risk premium of BAX for future research.

3 Impact of central bank announcements on uncertainty

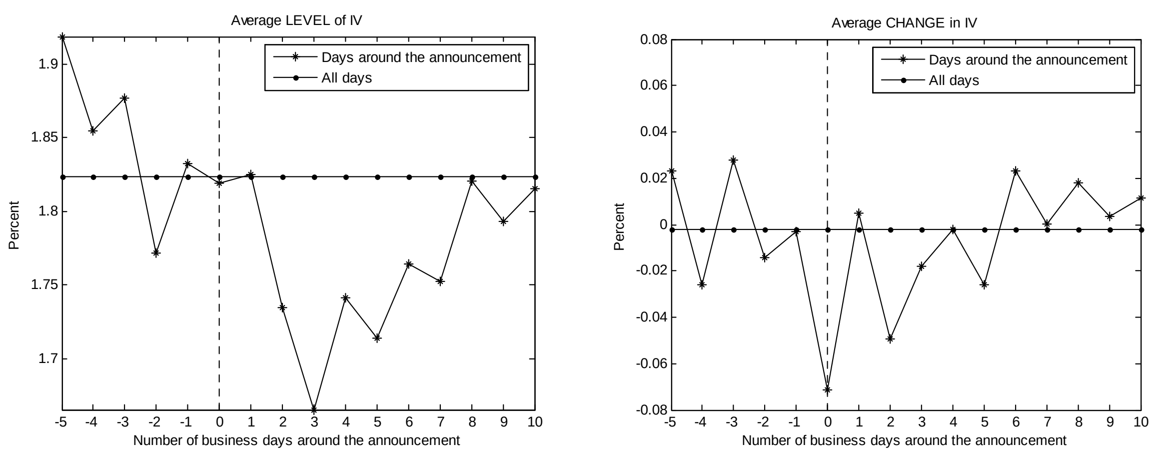
3.1 BAX IV around BoC target rate decisions

By far the most important announcements made by the central banks are those of target rate decisions. The current practice of target rate announcements adapted by most central banks is to make the announcements only on pre-scheduled dates. In Canada, these announcements, called Fixed Announcement Dates (FADs), occur every 6-8 weeks. The purpose of this approach is to reduce uncertainty about the timing of target rate changes in the market, a practice consistent with increased transparency in central bank communications.

After each target rate announcement, central banks are interested in assessing the impact of their decision on the market. Typically, the financial variables that they look at include yields on various fixed income securities and foreign exchange rates. These variables, however, do not tell us whether a particular decision has increased or decreased uncertainty around future target rates, which is also of interest to central bankers. Most of the time, the central bank aims to reduce uncertainty. In this section, we test whether the BoC has achieved this goal by looking at changes in our two proposed measures of uncertainty around target rate announcements.

We first look at whether any pattern emerges for BAX IV around target rate decision days. It is important to remember that the maturities of options used in our computation of IV range between one and six months, with an average maturity of three months. Since the underlying asset of OBX is BAX, which is the futures contract on the 3-month CDOR, our IV reflects uncertainty about the target rate approximately six month ahead. The fact that the target rate is fixed until the next target rate announcement day, which is typically 6-8 weeks away, means that we would expect any measure of uncertainty over the horizon that ends before the following target rate announcement to drop sharply after each announcement. However, if the horizon covered by the uncertainty measure extends beyond the following announcement day, which is the case for our BAX IV, uncertainty can either increase or decrease after a target rate announcement.

In <Figure 3>, we plot the average level of IV and average daily change in IV between -5 and +10 business days (equivalently between -1 and +2 weeks) around target rate announcements. We find that the average IV drops significantly on the announcement days (by 7 bps) and 2 days after the announcement days (by 5 bps), resulting in lower than average level of IV following target rate announcements. The effect is not permanent, however, and the IV reverts back to the average level after about 7 business days.



<Figure 3> Average level and change of IV around FADs

We test whether the observed drop in IV on target rate announcement days is statistically significant by running the following regression for each event day, $i \in [-5, +10]$.

$$\Delta IV_t = \alpha_i + \beta_i \cdot \mathrm{I}_i(t) + \varepsilon_{it}$$

where ΔIV_t is the daily change in IV on date t, and $\mathrm{I}_i(t)$ is an indicator function that yields 1 if date t is i business days away from a FAD and 0 otherwise. The intercept coefficient,

α_i, of this regression is then the average daily change in IV on all days other than the event days, i. The slope coefficient, β_i, is the deviation of daily change in IV on event days, i, compared to all other days. The regression results are reported in <Table 3>.

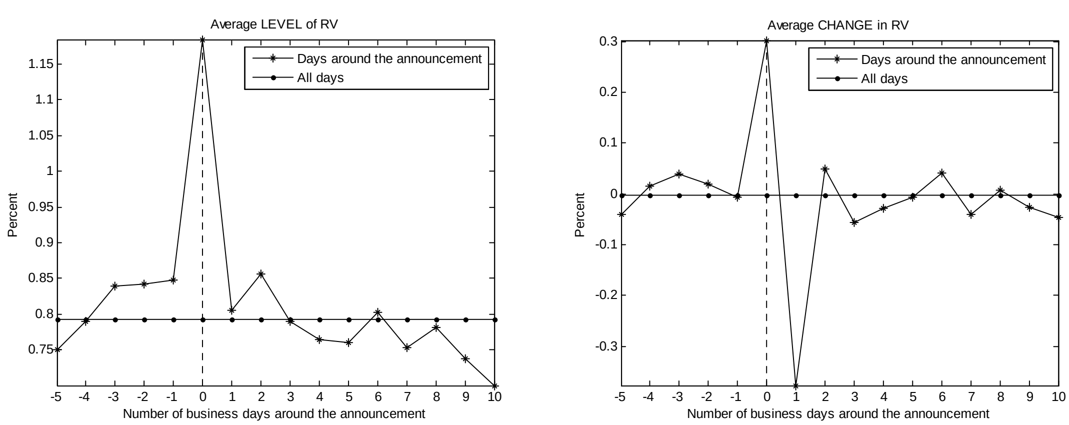
<Table 3: Change in IV and RV around the FADs>

We find that α_i, the average daily change in IV, is not statistically different from zero. We also find that statistically significant drops in IV occur on the day of the announcement ($i = 0$) and two days after the announcement ($i = +2$). The decrease in IV two days after the announcements can be explained by the fact that the BoC releases its Monetary Policy Reports (MPRs) two days after target rate announcements on every other FADs.

The results in this section shows that on average, the BoC target rate decisions succeeded in reducing uncertainty around future target rates in our sample period. A significant reduction in uncertainty is observed on both the target rate announcements and releases of Monetary Policy Reports, and the effect of the reduction in uncertainty lasts for about 7 business days.

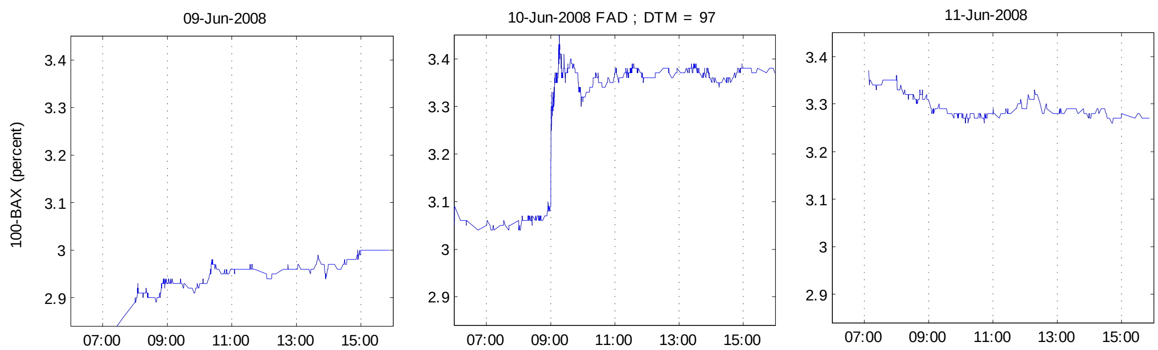
3.2 BAX RV around BoC target rate decisions

We investigate whether a similar conclusion can be reached using BAX RV. <Figure 4> shows the average level of RV and average daily change in RV between -5 and +10 business days around target rate announcements. Unlike IV, RV increases dramatically on the announcement days, but reverts back to its average level the day after.



<Figure 4> Average level and change of RV around FADs

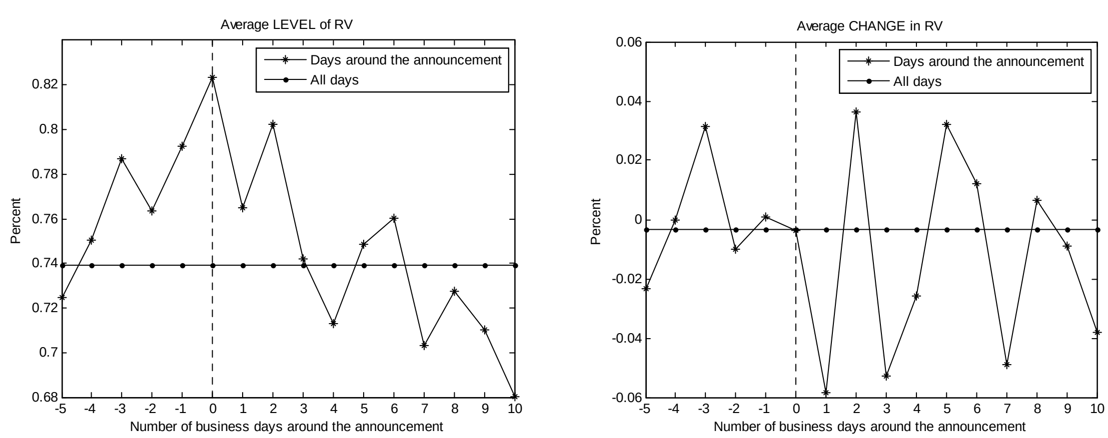
This behavior of RV around target rate decisions can be explained by the way intraday BAX prices behave on the FADs. <Figure 5> shows an example of typical behavior of intraday BAX prices on a FAD, together with the day before and after the FAD. Following most of the target rate announcements at 9 AM on the FADs, BAX price jumps to a new level. Since RV is computed by summing intraday price changes, this price jump results in a jump in RV, often dominating price variations in the rest of the day.



<Figure 5> Intraday BAX prices around a FAD

In order to remove the effect of a price jump immediately following target rate announcements, we recompute RVs on all days excluding prices between 8:30 AM and 9:30 AM. The results on event days based on the new RVs are plotted in <Figure 6>. With the new RVs, we still observe higher than average RVs on the FADs. However, the increase is not as large as with the old RVs. In fact, the average change in RV on the day of the FADs is close to the sample average because RV starts increasing a few days before the actual FADs. RV reaches its peak on the FADs, then decreases gradually following the FADs.

The results of the regressions on change in RV, which are reported in <Table 3>, show that a significant drop in RV is observed 1 and 3 business days after the FADs.



<Figure 6> Average level and change of RV around FADs (jump removed)

Although at first glance, the results based on IV and RV are somewhat different, both measures decrease following the FADs, indicating reduced uncertainty. The behavior of the two measures are different in that IV decreases right away on the announcement days whereas RV decreases more gradually on the following days. The discrepancy can be explained by the fact that IV is an expectation of future RVs. Therefore, any foreseen decrease in RV in a near future will be reflected in IV right away whereas the actual decrease in RV can be only observed on the future days.

In most cases, central banks are interested in the impact of their target rate decisions on uncertainty around the target rate that will prevail at some time in the future. Hence, when the goal is to assess impact of a particular event on uncertainty, IV, which is an expectation of uncertainty between today and a future day is more relevant than RV, a measure of uncertainty of today only. For this reason, we use IV rather than RV in our analyses of different events in the next section.

3.3 Impact of BoC policy actions during the financial crisis of 2007-2008 on uncertainty

A crisis period provides valuable examples in which the objective of a central bank's policy action is to reduce uncertainty in the market. During the financial crisis of 2007-2008, the BoC put several important policy actions into effect as did other central banks around the world. In this section, we examine the impact of these policy actions on uncertainty as reflected in changes in BAX IV on the day of the announcements.

We look at four important crisis-related policy announcements by the BoC between 2007 and 2010, and report the results in Panel A of <Table 4>. The policy actions considered include the introduction of term purchase and resale agreements (PRAs), unscheduled cut in the target rate coordinated with other major central banks, and conditional commitment of 2009-2010. We find that all of these announcements led to a large decrease in IV, ranging between -12 and -51 bps. By far the largest drop in IV of 51 bp occurred on October 8, 2008 when the target rate was cut by 75 bps in conjunction with other central banks This result is consistent with Bauer (2012) which finds that many of the important announcements by the Federal Reserve Board during the crisis also led to a larger than average drop in IV of the eurodollar futures options.

<Table 4: Important Bank of Canada policy actions between 2007 and 2012>

We also examine the impact of recent FAD announcements when the BoC introduced a shift in bias regarding the path of monetary policy in the near future. We expect such announcements to be associated with a larger than average change in IV on the day of the announcement. A priori, it is unclear whether these announcements will lead to an increase or a decrease in IV. Somewhat surprisingly, the results reported in panel B of <Table 4> show that all four FAD announcements which introduced a shift in bias between 2011 and 2012 led to a large drop in IV, indicating that the announcements succeeded in reducing uncertainty.

The analyses in this section show that IV is a better measure of uncertainty than RV for the purpose of assessing impact of announcements on uncertainty. However, there are other applications of IV and RV as measures of uncertainty where RV is as valuable as IV. We explore one such application in the next section.

4 Relation of uncertainty measures to OIS excess return

4.1 Forecasting magnitude of OIS excess returns

In this section, we test whether BAX IV and RV are positively related to ex-post realized uncertainty around future target rates. Realized uncertainty is measured by the magnitude of realized prediction error of OIS rate, which is the most widely used measure of target rate expectation in Canada.

In the absence of risk premium, OIS rates reflect today's expectation of average CORRA rates for a fixed term in the future. On the OIS contract's settlement date, the two parties in the OIS transaction exchange the difference between the realized CORRAs and the expected CORRAs (i.e., initial OIS rate). On average, the magnitude of the difference between the expected and realized CORRAs will be larger when uncertainty on the level of CORRA is higher.

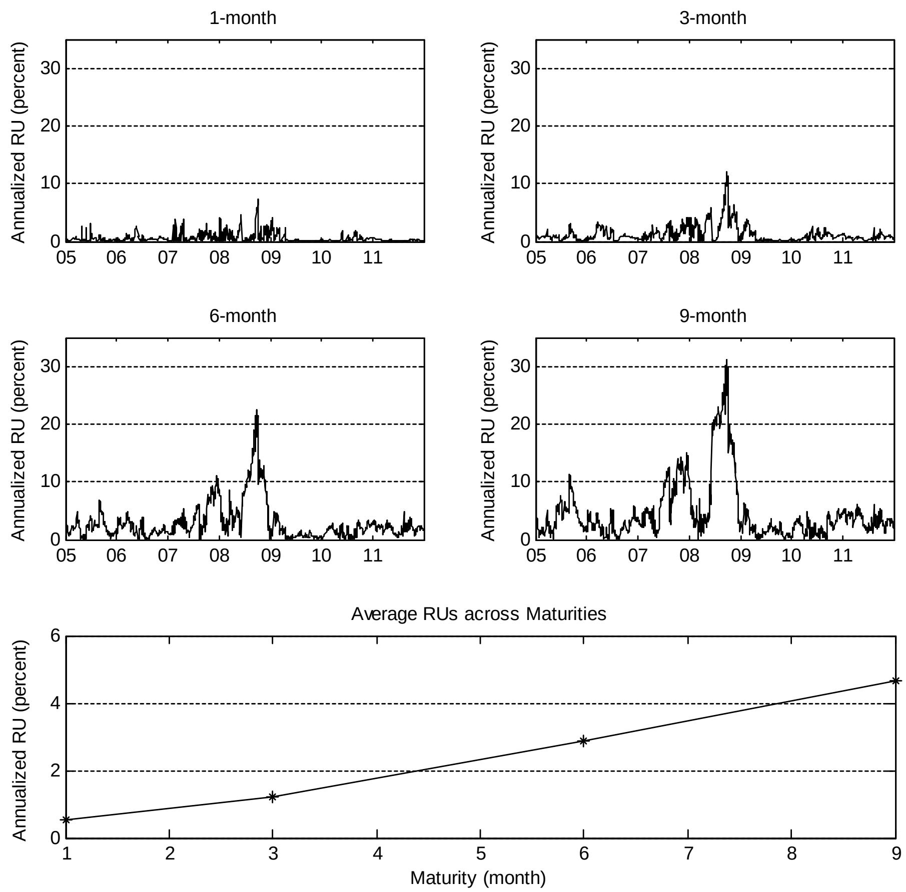
Let $OIS_{t,t+n}$ denote the OIS rate for month $t+n$ as quoted at the end of month t. We will refer to $n = 1$ as the one-month-ahead contract, $n = 2$ as the two-month-ahead contract, and so on. Let $CORRA_{t+1,t+n}$ denote the geometric mean of daily realized CORRAs between months $t+1$ and $t+n$. We define the n-month ahead realized uncertainty, $RU_{OIS}(t, t+n)$, as:

$$RU_{OIS}(t, t+n) \equiv |OIS_{t,t+n} - CORRA_{t+1,t+n}|, \quad (2)$$

where

$$CORRA_{t+1,t+n} = \left(\prod_{i=t+1}^{t+n} \left(1 + \frac{CORRA_i}{365} \right) - 1 \right) \frac{365}{n}.$$

We annualize RU_{OIS} through multiplying it by $\sqrt{365}$. We plot annualized RUs over 1, 3, 6, and 9 month horizons in <Figure 7>. RU_{OIS} increases with maturity, and displays significant variation across the years. RU_{OIS} reached its highest level in late 2008 in the middle of the US subprime crisis, and its lowest level between 2009 and 2010 when the target rate was kept constant at 0.25% following the conditional commitment of the BoC.



<Figure 7> Realized uncertainty in overnight rate

We now regress RU_{OIS} on BAX IV and RV to see if they are indeed positively related.

$$RU_{OIS}(t, t+n) = \alpha + \beta \cdot IV_t + \varepsilon_t$$
$$RU_{OIS}(t, t+n) = \alpha + \beta \cdot RV_t + \varepsilon_t.$$

where IV_t and RV_t are volatility measures observed on the last day of month t. We first run univariate regressions with each volatility measure separately, then run another regression including both IV and RV as regressors. The results reported in <Table 5> show that the coefficients on IV and RV are positive in all the regressions. The coefficients are also significant at 90% confidence level in all the univariate regressions. The adjusted R^2s of the multivariate regressions range from 0.17 to 0.46, indicating that IV and RV can predict a large portion of variation in our measure of realized uncertainty.

<Table 5: Forecasting realized uncertainty>

The analysis shows that BAX IV and RV are good measures of uncertainty around future target rates. In the following two sections, we apply these two volatility measures to situations that could be of interest to monetary policy decision makers.

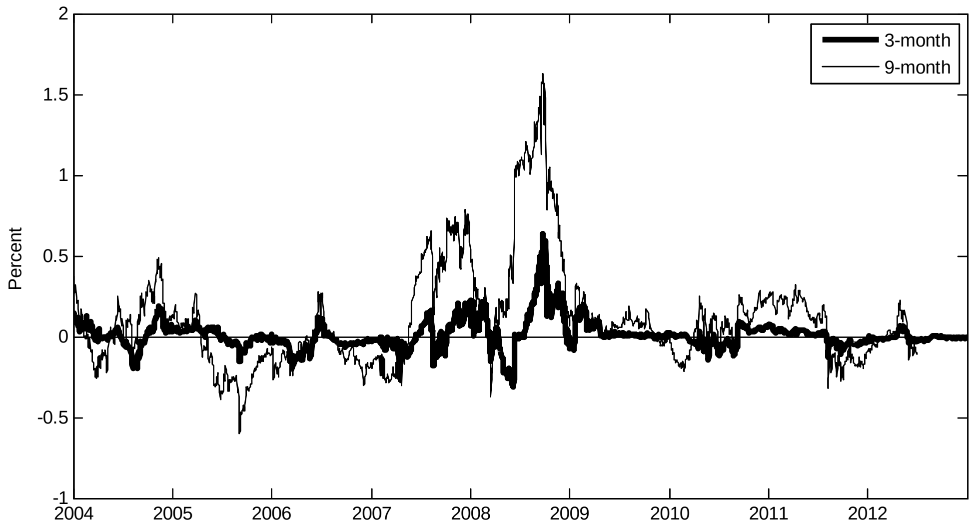
4.2 Forecasting risk premium in the OIS market

A measure of uncertainty around future target rates can be also useful in refining existing methodologies of extracting the market's expectation of future target rates. In the US, federal funds futures are used to obtain the market's expectation. In Canada, however, the futures contracts equivalent to the federal funds futures have very low liquidity, so overnight indexed swap (OIS) rates are used instead.

Piazzesi and Swanson (2008) show that although federal funds futures provide good forecasts of future target rates, the extracted forecasts need to be adjusted to account for risk premium in the federal funds futures prices. They find that business-cycle indicators such as employment growth, corporate bond spread, and treasury yield spread are good predictors of the risk premium, thus can be used to improve prediction.

We propose using BAX IV and RV as additional predictors to the ones suggested by Piazzesi and Swanson (2008), in forecasting risk premium in OIS rates. Our proposal is based on the intuition that investors will demand higher risk premium in the OIS market when uncertainty around future target rates is higher.

We first plot the time series of prediction error of OIS rates to see if such risk premium exists. <Figure 8> shows the OIS prediction error, or OIS excess return, for OIS contracts with 3-month and 9-month maturities. We find that OIS excess returns are persistent, and can be both positive and negative. OIS excess returns are also greater in magnitude for longer maturities. The fact that OIS excess returns are persistent implies that the returns are likely to be predictable.



<Figure 8> OIS Excess Returns

4.2.1 In-sample tests

We now test whether BAX IV and RV can predict OIS excess returns by running univariate regressions of the form,

$$
\begin{aligned}
R_{OIS}(t, t+n) &= \alpha + \beta \cdot IV_t + \varepsilon_t \\
R_{OIS}(t, t+n) &= \alpha + \beta \cdot RV_t + \varepsilon_t.
\end{aligned}
$$

where IV_t and RV_t are the volatility measures on the last day of month t. $R_{OIS}(t, t+n)$ is the OIS excess return defined as,

$$R_{OIS}(t, t+n) \equiv OIS_{t,t+n} - CORRA_{t+1,t+h},$$

where $OIS_{t,t+n}$ is the n-month maturity OIS rate observed on the last day of month t, and $CORRA_{t+1,t+n}$ denotes the geometric mean of daily realized CORRAs between months $t+1$ and $t+n$,

$$CORRA_{t+1,t+n} = \left(\prod_{i=t+1}^{t+n} \left(1 + \frac{CORRA_i}{365} \right) - 1 \right) \frac{365}{n}.$$

Note that the realized uncertainty measure we introduced in equation (2) is simply the absolute value of OIS excess return, R_{OIS}.

For comparison, we run the same univariate regression on the three business-cycle variables in Piazzesi and Swanson (2008): (i) the spread between BBB-rated 10-year corporate bonds and the 10-year Treasury yield, (ii) the spread between 2-year and 5-year Treasury yields, and (iii) employment growth. We also consider the index of Canadian economic policy uncertainty based on Baker, Bloom, and Davis (2013) as well as lagged OIS excess return, $R_{OIS}\,(t-3,t)$, and lagged realized uncertainty, $RU_{OIS}\,(t-3,t)$. We use the 3-month lag because we want the lag to be close enough to the current date without being too short since one to two month maturity often has relatively little uncertainty.

We report the correlations of all the predictors, excluding the lagged R_{OIS} and RU_{OIS}, in <Table 6>. All the predictors are positively correlated except for employment growth, which is negatively correlated to all the other predictors. The negative correlation of employment growth to the other variables is consistent with the fact that high asset price volatility, high corporate bond spread, and high policy uncertainty are all linked to bad states of the economy whereas high employment growth is linked to good states of the economy.

<Table 6: Correlations>

We report the results of the univariate regressions in <Table 7>. We find that all slope coefficients for IV and RV are positive except for IV at 1-month maturity. This confirms our intuition that the risk premium must be positively related to uncertainty. RV performs better than IV in terms of both R^2 and t-statistic. The regressions with RV exhibit higher coefficients and R^2. Moreover, the coefficients on RV are highly significant at all maturities longer than 1-month whereas the coefficients on IV are significant only at 9-month maturity.

<Table 7: Forecasting OIS excess returns - univariate regressions>

Next, we test whether BAX IV and RV have additional predictive power when the business-cycle indicators, policy uncertainty measure, lagged R_{OIS}, and RU_{OIS} are also used as predictors. The results of the multivariate regression including all predictors as regressors are reported in <Table 8>.

<Table 8: Forecasting OIS excess returns - multivariate regressions>

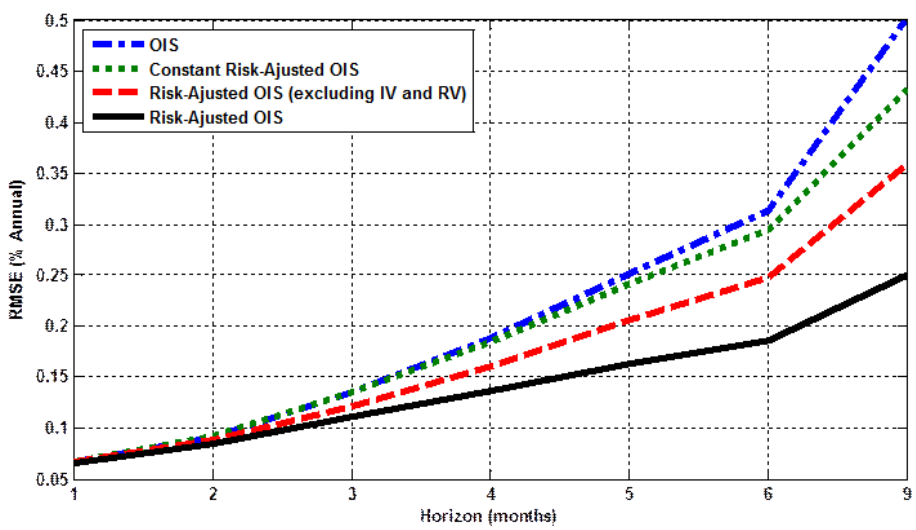
We find that adding IV and RV as regressors improve adjusted R^2 of the regression for all maturities. The improvement in adjusted R^2 ranges from 0.01 for 1-month maturity to 0.12 for 9-month maturity. We find that in general the magnitude of the improvement increases with forecasting horizon. We also observe that the significance of IV improves in the multivariate regression compared to the univariate regression whereas the opposite is true for RV.

4.2.2 Out-of-sample tests

The regression results presented in the previous section are in-sample tests of forecasting ability of IV and RV. We now present the results of out-of-sample tests. In <Figure 9>, we plot the root-mean-squared-error (RMSE) of four different predictions of realized CORRAs. We compare the performance of (i) unadjusted OIS, (ii) constant risk-adjusted OIS, (iii) risk-adjusted OIS using predictors excluding IV and RV, and (iv) risk-adjusted OIS including all the predictors.

The out-of-the sample predictions are conducted daily starting from January 2007, which is one year after our first data point in IV. We adjust the OIS-implied CORRAs on the first day of 2007 by using the coefficients obtained from the multivariate regression with all the predictors based on daily times series of the predictors in 2006. For the adjustment on the second day of 2007, we use the coefficients estimated from running the regression on data in the expanded window from January 2006 up to the first day of 2007. The procedure is repeated until the end of our sample.

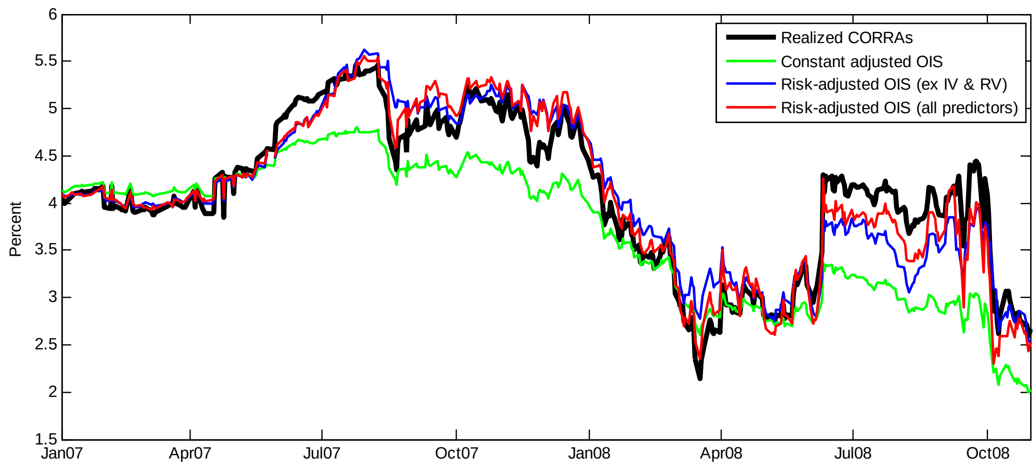
We find that improvement in RMSE is negligible in short maturities of 1-3 month, but is substantial at longer maturities such as 6-month or 9-month. If we use all the predictors (black line), improvement over unadjusted OIS prediction (blue line) is around 10 bps for 6-month maturity and 25 bps for 9-month maturity.



<Figure 9> Out-of-sample RMSE of daily OIS excess return prediction

<Figure 10> compares out-of-sample forecasts of CORRAs over a 9-month horizon using three different models: (i) constant adjusted OIS, (ii) risk-adjusted OIS using all predictors

except IV and RV, and (iii) risk-adjusted OIS using all predictors. Different models performed similarly in the later part of the sample in which the target rate stayed at 1% most of the time, so we only plot earlier period between January 2007 and October 2008. Prediction improves substantially when we adjust OIS rates using economic variables as predictors. Adding IV and RV to the regression improves forecasts slightly especially in 2008.



<Figure 10> Risk-adjusted OIS rates

The results in this section show that BAX IV and RV are closely related to risk premium in the OIS market, making these measures useful for adjusting OIS-implied expectations of future target rates. The resulting risk-adjusted OIS rates are closer to realized CORRAs than unadjusted OIS rates, providing substantial improvement in prediction.

5 Conclusion

We show that implied volatility computed from interest rate futures options and realized volatility computed from intraday prices of interest rate futures are useful indicators of uncertainty around future central bank target interest rates. Based on implied volatility and realized volatility computed from BAX futures and OBX options in Canada, we show that on average the target rate announcements by the Bank of Canada reduced uncertainty around future target rates. We also find that some of the most important policy actions taken by the Bank of Canada as a response to the financial crisis of 2007-2008, including the conditional commitment of 2009-2010, unscheduled cut in the target rate coordinated with other major central banks, and introduction of term purchase and resale agreements (PRAs), all reduced uncertainty.

We also explore the relation between our two measures of uncertainty and the risk premium in the OIS market. We find that implied volatility and realized volatility of BAX are positively related to risk premium in OIS rates, thus can be used to adjust OIS-implied expectations of future target rates for a risk premium. The resulting risk-adjusted OIS rates are closer to realized CORRAs than unadjusted OIS rates, providing improvement in prediction of future target rates.

Appendix A. computing implied volatility of BAX

We assume that the short rate, r_t, follows the risk-neutral process proposed in Vasicek (1977).

$$dr_t = a\,(b - r)\,dt + \sigma dz.$$

Yield formula

Vasicek (1977) shows that the yield to maturity h under his model is

$$R\,(t,h) = -\frac{1}{h}\ln A\,(h) + \frac{1}{h}B\,(h)\,r_t,$$

where

$$B\,(h) = \frac{1 - e^{-ah}}{a}$$

and

$$A\,(h) = \exp\left(\frac{(B\,(h) - h)\left(a^2 b - \frac{\sigma^2}{2}\right)}{a^2} - \frac{\sigma^2 B\,(h)^2}{4a}\right).$$

Futures formula

The price of a futures contract on yield $R\,(t+h_1,h_2)$ is

$$\begin{aligned} f\,(t,h_1,h_2) &= E_t\left[\exp\left(-\int_t^{t+h_1} r_s d_s\right) R\,(t+h_1,h_2)\right] \\ &= E_t\left[\exp\left(-\int_t^{t+h_1} r_s d_s\right)\left(-\frac{1}{h_2}\ln A\,(h_2) + \frac{1}{h_2}B\,(h_2)\,r_{t+h_1}\right)\right] \\ &= -\frac{1}{h_2}\ln A\,(h_2)\exp\left(-h_1 R\,(t,h_1)\right) + \frac{1}{h_2}B\,(h_2)\,E_t\left[\exp\left(-\int_t^{t+h_1} r_s d_s\right) r_{t+h_1}\right]. \end{aligned}$$

Let

$$V\,(h,r) \equiv E_t\left[\exp\left(-\int_t^{t+h} r_s d_s\right) r_{t+h}\right].$$

V is the solution to the Feynman-Kac PDE and the boundary condition,

$$-V_h + \frac{1}{2}\sigma^2 V_{rr} + a(b-r)V_r - rV = 0, \quad V(0,r) = r. \tag{3}$$

We conjecture that

$$V(h,r) = (\alpha(h) + \beta(h)r)\exp(\gamma(h)r).$$

Then we have

$$\begin{aligned}
V_h &= \left(\dot{\alpha} + \left(\dot{\beta} + \alpha\dot{\gamma}\right)r + \beta\dot{\gamma}r^2\right)\exp(\gamma r) \\
V_r &= (\alpha\gamma + \beta + \beta\gamma r)\exp(\gamma r) \\
V_{rr} &= \left(2\beta\gamma + \alpha\gamma^2 + \beta\gamma^2 r\right)\exp(\gamma r) \\
\gamma(0) &= \alpha(0) = 0,\ \beta(0) = 1.
\end{aligned}$$

Plugging these expressions into equation (3), we get

$$-\left(\dot{\alpha} + \left(\dot{\beta} + \alpha\dot{\gamma}\right)r + \beta\dot{\gamma}r^2\right) + \frac{1}{2}\sigma^2\left(2\beta\gamma + \alpha\gamma^2 + \beta\gamma^2 r\right) + a(b-r)(\alpha\gamma + \beta + \beta\gamma r) - r(\alpha + \beta r) = 0.$$

Hence,

$$\begin{cases}
-\dot{\alpha} + \frac{1}{2}\sigma^2\left(2\beta\gamma + \alpha\gamma^2\right) + ab(\alpha\gamma + \beta) = 0 \\
-\left(\dot{\beta} + \alpha\dot{\gamma}\right) + \frac{1}{2}\sigma^2\beta\gamma^2 + ab\beta\gamma - a(\alpha\gamma + \beta) - \alpha = 0 \\
-\beta\dot{\gamma} - a\beta\gamma - \beta = 0
\end{cases}.$$

The last equation implies that

$$\gamma(h) = -B(h) = -\frac{1 - e^{-ah}}{a}, \tag{4}$$

and the second equation can be rewritten as

$$\left(\frac{1}{2}\sigma^2\gamma^2 + ab\gamma - a\right)\beta = \dot{\beta}. \tag{5}$$

From equations (4) and (5), we get

$$\begin{aligned}
\beta(h) &= \exp\left(\left(ba^2 - \frac{\sigma^2}{2}\right)\frac{(B(h) - h)}{a^2} - ah - \frac{\sigma^2 B(h)^2}{4a}\right) \\
&= e^{-ah}A(h) = (1 - aB(h))A(h).
\end{aligned}$$

The first equation implies that

$$\left(\sigma^2\gamma + ab\right)\beta + \left(\frac{1}{2}\sigma^2\gamma^2 + ab\gamma\right)\alpha = \dot{\alpha} \tag{6}$$

Now we look for a particular solution of type

$$\alpha = \left(k_1 B\left(h\right) + k_2\right)\beta. \tag{7}$$

The derivative of α is then

$$\begin{aligned}
\dot{\alpha} &= k_1 \dot{B}\left(h\right)\beta + \left(k_1 B\left(h\right) + k_2\right)\dot{\beta} \\
&= k_1\left(1 - aB\left(h\right)\right)\beta + \left(k_1 B\left(h\right) + k_2\right)\left(\frac{1}{2}\sigma^2 B\left(h\right)^2 - abB\left(h\right) - a\right)\beta \\
&= \left[k_1\left(1 - aB\left(h\right)\right) + \left(k_1 B\left(h\right) + k_2\right)\left(\frac{1}{2}\sigma^2 B\left(h\right)^2 - abB\left(h\right) - a\right)\right]\beta. \qquad (8)
\end{aligned}$$

Combining the equations (6), (7), and (8), we get

$$k_1\left(1 - aB\left(h\right)\right) - a\left(k_1 B\left(h\right) + k_2\right) + \sigma^2 B\left(h\right) - ab = 0. \tag{9}$$

This equation holds if we set k_1 and k_2 to be

$$\begin{aligned}
k_1 &= \frac{\sigma^2}{2a} \\
k_2 &= \frac{\sigma^2}{2a^2} - b.
\end{aligned}$$

Hence, the general solution is

$$\alpha\left(h\right) = kA\left(h\right) + \left(\frac{\sigma^2}{2a}B\left(h\right) + \frac{\sigma^2}{2a^2} - b\right)e^{-ah}A\left(h\right).$$

The boundary condition, $\alpha\left(0\right) = 0$, implies that

$$k = b - \frac{\sigma^2}{2a^2},$$

so that

$$\begin{aligned}
\alpha\left(h\right) &= \left(\frac{\sigma^2}{2a}e^{-ah}B\left(h\right) + \left(\frac{\sigma^2}{2a^2} - b\right)e^{-ah} + b - \frac{\sigma^2}{2a^2}\right)A\left(h\right) \\
&= \left(\frac{\sigma^2}{2a}e^{-ah}B\left(h\right) + \left(b - \frac{\sigma^2}{2a^2}\right)\left(1 - e^{-ah}\right)\right)A\left(h\right) \\
&= \left(\frac{\sigma^2}{2a}e^{-ah}B\left(h\right) + a\left(b - \frac{\sigma^2}{2a^2}\right)B\left(h\right)\right)A\left(h\right) \\
&= \left(\frac{\sigma^2}{2a}\left(e^{-ah} - 1 + 1\right)B\left(h\right) + a\left(b - \frac{\sigma^2}{2a^2}\right)B\left(h\right)\right)A\left(h\right) \\
&= \left(\frac{\sigma^2}{2a}\left(-aB\left(h\right)^2 + B\left(h\right)\right) + \left(ab - \frac{\sigma^2}{2a}\right)B\left(h\right)\right)A\left(h\right) \\
&= \left(ab - \frac{\sigma^2}{2}B\left(h\right)\right)B\left(h\right)A\left(h\right).
\end{aligned}$$

Therefore, the futures price is,

$$\begin{aligned} f(t,h_1,h_2) &= \left[-\frac{1}{h_2}A(h_1)\ln A(h_2) + \frac{1}{h_2}B(h_2)\alpha(h_1) + \frac{1}{h_2}B(h_2)\beta(h_1)r_t\right]\exp(-B(h_1)r_t) \\ &\equiv (C(h_1,h_2) + D(h_1,h_2)r_t)\exp(-B(h_1)r_t) \end{aligned}$$

Integrated variance of the futures and its expectation

We apply Ito's lemma on $f(t,h_1,h_2)$ to get

$$df = \left[f_t + a(b-r)f_r + \frac{\sigma^2}{2}f_{rr}\right]dt + \sigma f_r dz$$

and

$$\begin{aligned} f_r &= [D(h_1,h_2) - B(h_1)C(h_1,h_2) - B(h_1)D(h_1,h_2)r_t]\exp(-B(h_1)r_t) \\ &\equiv (E(h_1,h_2) + F(h_1,h_2)r_t)\exp(-B(h_1)r_t). \end{aligned}$$

The integrated variance of $f(t,h_1,h_2)$ between t and $t+h$ with $h \leq h_1$ is defined as

$$\begin{aligned} IV^f(t,t+h) &= \sigma^2\int_t^{t+h} f_{r_s}^2 d_s \\ &= \sigma^2\int_t^{t+h}(E(h_1,h_2) + F(h_1,h_2)r_s)^2\exp(-2B(h_1)r_s)d_s. \end{aligned}$$

The expectation of the integrated variance of $f(t,h_1,h_2)$ is

$$E_t\left[IV^f(t,t+h)\right] = \sigma^2 V(h,r_t), \tag{10}$$

$$\begin{aligned} V(h,r_t) &= \int_t^{t+h} E_t\left[(E(h_1,h_2) + F(h_1,h_2)r_s)^2\exp(-2B(h_1)r_s)\right]d_s \\ &= \int_0^h E_t\left[(E(h_1,h_2) + F(h_1,h_2)r_{t+s})^2\exp(-2B(h_1)r_{t+s})\right]d_s, \end{aligned}$$

where

$$E_t\left[(\omega+\rho r_{t+s})^2\exp(-ur_{t+s})\right] = \omega^2 E_t[\exp(-ur_{t+s})] + 2\omega\rho E_t[r_{t+s}\exp(-ur_{t+s})] + \rho^2 E_t\left[r_{t+s}^2\exp(-ur_{t+s})\right],$$

$$E_t[\exp(-ur_{t+s})] = \exp(\mu(u,s) + \phi(u,s)r_t),$$

and where

$$\begin{aligned}\mu(u,s) &= \frac{\sigma^2}{2a}\left(1-e^{-2as}\right)\frac{u^2}{2} - B(s)\,abu \\ &= \frac{\sigma^2 u^2}{4}B(2s) - B(s)\,abu \\ \phi(u,s) &= -ue^{-as}.\end{aligned}$$

Since

$$\begin{aligned}E_t\left[r_{t+s}\exp\left(-ur_{t+s}\right)\right] &= -\left(\frac{\sigma^2}{2}B(2s)u - B(s)\,ab - e^{-as}r_t\right)E_t\left[\exp\left(-ur_{t+s}\right)\right], \\ E_t\left[r_{t+s}^2\exp\left(-ur_{t+s}\right)\right] &= \left(B(2s)\frac{\sigma^2}{2}\right)E_t\left[\exp\left(-ur_{t+s}\right)\right] + \\ &\quad \left(\frac{\sigma^2}{2}B(2s)u - B(s)\,ab - e^{-as}r_t\right)^2 E_t\left[\exp\left(-ur_{t+s}\right)\right],\end{aligned}$$

we get,

$$\begin{aligned}&E_t\left[\left(\omega+\rho r_{t+s}\right)^2\exp\left(-ur_{t+s}\right)\right] \\ =& \left[\begin{array}{c}\omega^2 - 2\omega\rho\left(\frac{\sigma^2}{2}B(2s)u - B(s)\,ab - e^{-as}r_t\right) + \frac{\rho^2\sigma^2}{2}B(2s) \\ +\rho^2\left(\frac{\sigma^2}{2}B(2s)u - B(s)\,ab - e^{-as}r_t\right)^2\end{array}\right]E_t\left[\exp\left(-ur_{t+s}\right)\right] \\ =& \left[\left(\rho\left(\frac{\sigma^2}{2}B(2s)u - B(s)\,ab - e^{-as}r_t\right) - \omega\right)^2 + \frac{\rho^2\sigma^2}{2}B(2s)\right]E_t\left[\exp\left(-ur_{t+s}\right)\right].\end{aligned}$$

Price of European options on futures

The price of a European call option on the futures, $f(t,h_1,h_2)$, with maturity h and strike $\bar{f}$ is

$$\begin{aligned}&C(t,h,h_1,h_2) \\ =& E_t\left[\exp\left(-\int_t^{t+h} r_s d_s\right)\left(f(t+h)-\bar{f}\right)1_{[f>\bar{f}]}\right] \\ =& E_t\left[E_t\left[\exp\left(-\int_t^{t+h} r_s d_s\right)\middle| r_{t+h}\right]\left(f(t+h)-\bar{f}\right)1_{[f>\bar{f}]}\right].\end{aligned}$$

Joint distribution of r_{t+h} and $\int_t^{t+h} r_s d_s$

One of the ways to characterize the joint distribution of r_{t+h} and $\int_t^{t+h} r_s d_s$ is through the moment generating function,

$$V(h,r) = E_t\left[\exp\left(-ur_{t+h} - v\int_t^{t+h} r_s d_s\right)\right].$$

V is the solution to the Feynman-Kac PDE and the boundary condition

$$\begin{aligned} -V_h + \frac{1}{2}\sigma^2 V_{rr} + a(b-r)V_r - vrV &= 0, \\ V(0,r) &= \exp(-ur) \end{aligned} \tag{11}$$

We conjecture that

$$V(h,r) = \alpha(h)\exp(\gamma(h)r).$$

Then we have,

$$\begin{aligned} V_h &= (\dot{\alpha} + \alpha\dot{\gamma}r)\exp(\gamma r) \\ V_r &= \alpha\gamma\exp(\gamma r) \\ a(b-r)V_r &= (ab\alpha\gamma - a\alpha\gamma r)\exp(\gamma r) \\ V_{rr} &= \alpha\gamma^2\exp(\gamma r) \\ \gamma(0) &= -u, \alpha(0) = 1. \end{aligned}$$

Plugging these expressions into equation (11) yields

$$\begin{aligned} -\dot{\alpha} - \alpha\dot{\gamma}r + \frac{1}{2}\sigma^2\alpha\gamma^2 + ab\alpha\gamma - a\alpha\gamma r - v\alpha r &= 0 \\ -\dot{\alpha} + \frac{1}{2}\sigma^2\alpha\gamma^2 + ab\alpha\gamma &= 0 \\ -\dot{\gamma} - a\gamma - v &= 0 \end{aligned}$$

Hence,

$$\begin{aligned} \gamma &= -e^{-ah}u - B(h)v \\ \alpha &= \exp\left(\begin{array}{c} ab\left(-B(h)u - \frac{h-B(h)}{a}v\right) \\ +\frac{\sigma^2}{2}\left(\frac{u^2}{2}B(2h) + \frac{v^2}{a^2}\left(h - 2B(h) + \frac{B(2h)}{2}\right) + \frac{2uv}{a}\left(B(h) - \frac{B(2h)}{2}\right)\right) \end{array}\right) \\ &= \exp\left(\begin{array}{c} -abB(h)u - b(h-B(h))v \\ +\frac{\sigma^2 B(2h)}{4}u^2 + \frac{\sigma^2}{a^2}\left(\frac{h}{2} - B(h) + \frac{B(2h)}{4}\right)v^2 + \frac{\sigma^2}{a}\left(B(h) - \frac{B(2h)}{2}\right)uv \end{array}\right), \end{aligned}$$

and

$$\begin{aligned} & E_t\left[\exp\left(-ur_{t+h} - v\int_t^{t+h} r_s d_s\right)\right] \\ = & \exp\left(\begin{array}{c} -\left(abB(h) + e^{-ah}r_t\right)u - \left(b(h-B(h)) + B(h)r_t\right)v \\ +\frac{\sigma^2 B(2h)}{4}u^2 + \frac{\sigma^2}{a^2}\left(\frac{h}{2} - B(h) + \frac{B(2h)}{4}\right)v^2 + \frac{\sigma^2}{a}\left(B(h) - \frac{B(2h)}{2}\right)uv \end{array}\right). \end{aligned}$$

Therefore, the joint distribution of r_{t+h} and $\int_t^{t+h} r_s d_s$ is

$$\begin{pmatrix} r_{t+h} \\ \int_t^{t+h} r_s d_s \end{pmatrix} \sim N\left(\begin{pmatrix} b+e^{-ah}\left(r_t-b\right) \\ bh+B\left(h\right)\left(r_t-b\right) \end{pmatrix}, \begin{bmatrix} \frac{\sigma^2 B(2h)}{2} & \frac{\sigma^2}{a}\left(B\left(h\right)-\frac{B(2h)}{2}\right) \\ \frac{\sigma^2}{a}\left(B\left(h\right)-\frac{B(2h)}{2}\right) & \frac{\sigma^2}{a^2}\left(h-2B(h)+\frac{B(2h)}{2}\right) \end{bmatrix}\right),$$

and the conditional distribution of $\int_t^{t+h} r_s d_s$ given r_{t+h} is

$$\begin{aligned} \int_t^{t+h} r_s d_s \Big| r_{t+h} &\sim N\left(\begin{array}{c} bh+\left(B\left(h\right)-e^{-ah}\frac{(2B(h)-B(2h))}{aB(2h)}\right)\left(r_t-b\right) \\ +\frac{(2B(h)-B(2h))}{aB(2h)}\left(r_{t+h}-b\right), \left(\frac{\sigma^2}{a^2}\frac{\frac{B(2h)}{2}\left(h-2B(h)+\frac{B(2h)}{2}\right)-\left(B(h)-\frac{B(2h)}{2}\right)^2}{\frac{B(2h)}{2}}\right) \end{array} \right) \\ &\sim N\left(\begin{array}{c} bh+\left(\frac{2aB(h)^2-2B(h)+B(2h)}{aB(2h)}\right)\left(r_t-b\right) \\ +\frac{(2B(h)-B(2h))}{aB(2h)}\left(r_{t+h}-b\right), \frac{\sigma^2}{a^2}\frac{hB(2h)-2B(h)^2}{B(2h)} \end{array} \right) \\ &\sim N\left(\begin{array}{c} bh+\frac{2B(h)-B(2h)}{aB(2h)}\left(r_t-b\right) \\ +\frac{(2B(h)-B(2h))}{aB(2h)}\left(r_{t+h}-b\right), \frac{\sigma^2}{a^2}\frac{hB(2h)-2B(h)^2}{B(2h)} \end{array} \right) \\ &\sim N\left(bh+\frac{\left(2B\left(h\right)-B\left(2h\right)\right)}{aB\left(2h\right)}\left(r_{t+h}-b+r_t-b\right), \frac{\sigma^2}{a^2}\frac{hB\left(2h\right)-2B\left(h\right)^2}{B\left(2h\right)}\right). \end{aligned}$$

Going back to option price

Given the expression for the joint distribution of r_{t+h} and $\int_t^{t+h} r_s d_s$, we can now compute the price of a European call option on futures as follows.

$$\begin{aligned} & C(t,h,h_1,h_2) \\ = \;& E_t\left[\exp\left(-\int_t^{t+h} r_s d_s\right)\left(f\left(t+h\right)-\bar{f}\right)1_{[f>\bar{f}]}\right] \\ = \;& E_t\left[E_t\left[\exp\left(-\int_t^{t+h} r_s d_s\right)\Big| r_{t+h}\right]\left(f\left(t+h\right)-\bar{f}\right)1_{[f>\bar{f}]}\right] \\ = \;& E_t\left[\begin{array}{c} \exp\left(-\left(bh+\frac{(2B(h)-B(2h))}{aB(2h)}\left(r_{t+h}-b+r_t-b\right)\right)+\frac{\sigma^2}{2a^2}\frac{hB(2h)-2B(h)^2}{B(2h)}\right) \\ \left(f\left(t+h\right)-\bar{f}\right)1_{[f>\bar{f}]} \end{array} \right] \\ = \;& \exp\left(-\left(bh+\frac{\left(2B\left(h\right)-B\left(2h\right)\right)}{aB\left(2h\right)}\left(r_t-b\right)\right)+\frac{\sigma^2}{2a^2}\frac{hB\left(2h\right)-2B\left(h\right)^2}{B\left(2h\right)}\right)\times \\ & E_t\left[\exp\left(-\frac{\left(2B\left(h\right)-B\left(2h\right)\right)}{aB\left(2h\right)}\left(r_{t+h}-b\right)\right)\left(f\left(t+h\right)-\bar{f}\right)1_{[f>\bar{f}]}\right], \end{aligned}$$

where

$$E_t\left[\exp\left(-\frac{(2B(h)-B(2h))}{aB(2h)}(r_{t+h}-b)\right)\left(f(t+h)-\bar{f}\right)1_{[f>\bar{f}]}\right]$$

$$= \frac{1}{\sigma\sqrt{\pi B(2h)}}\int_{-\infty}^{+\infty}\exp\left(\begin{array}{c}-\frac{(2B(h)-B(2h))}{aB(2h)}(r_{t+h}-b)\\ -\frac{\left(r_{t+h}-b-e^{-ah}(r_t-b)\right)^2}{\sigma^2B(2h)}\end{array}\right)\left(f(t+h)-\bar{f}\right)1_{[f>\bar{f}]}dr_{t+h}$$

$$= \frac{1}{\sigma\sqrt{\pi B(2h)}}\int_{-\infty}^{+\infty}\exp\left(\begin{array}{c}-\frac{(2B(h)-B(2h))}{aB(2h)}(r_{t+h}-b)\\ -\frac{\left(r_{t+h}-b-e^{-ah}(r_t-b)\right)^2}{\sigma^2B(2h)}\end{array}\right)\left(f(t+h)-\bar{f}\right)1_{[f>\bar{f}]}dr_{t+h}.$$

Estimation strategy

We use options that meet the following criteria:

- The option has either non-zero open interest or non-zero trade on a given day.
- The option's price is greater than the minimum price. For OBX, this is 1 basis point in quote which is equivalent to $25.
- The option's days-to-maturity is between 30 and 180 days.

The Vasicek (1977) model has three unknown parameters: a, b, and σ. We adopt a two-stage estimation procedure described below:

1. On the first stage, we use the option valuation errors defined as

   $$e_j = C_j^{Mkt} - C_j^{Mod},$$

   and apply the Gaussian log likelihood

   $$\ln L^O \propto -\frac{1}{2}\sum_{j=1}^{N}\left\{\ln\left(RMSE^2\right) + e_j^2/RMSE^2\right\}, \quad (12)$$

   where

   $$RMSE \equiv \sqrt{\frac{1}{N}\sum_{j=1}^{N} e_j^2}.$$

   Using options prices in the past one year period, we maximize this log-likelihood, $\ln L^O$, to compute the first-stage estimates, $\hat{a}$, $\hat{b}$, and $\hat{\sigma}$. N is the number of option contracts available.

2. On the second stage, we fix a and b to their first-stage estimates, then apply the Gaussian log likelihood

$$\ln L_t^O \propto -\frac{1}{2}\sum_{j=1}^{N_t}\left\{\ln\left(RMSE_t^2\right) + e_j^2/RMSE_t^2\right\}, \tag{13}$$

   where

$$RMSE_t \equiv \sqrt{\frac{1}{N_t}\sum_{j=1}^{N_t} e_j^2}.$$

   On each day, we maximize this log-likelihood, $\ln L_t^O$, to compute the second-stage estimate $\hat{\sigma}_t$. N_t is the number of option contracts available on day t. On this second stage, we fix the initial value of σ to its first-stage estimate $\hat{\sigma}$.

3. Finally, for a given horizon and a given day t, we use $\hat{a}$, $\hat{b}$, and $\hat{\sigma}_t$ to compute the Q-expectation of future integrated variance given in equation (10).

Appendix B. Computing realized volatility of BAX

Following Andersen and Bollerslev (1998), Barndorff-Nielsen and Shephard (2002), and Meddahi (2002), we compute the realized volatility of BAX as,

$$RV = \Delta S_{overnight}^2 + \sum_{i=1}^{N-1}\left(S_{i+1} - S_i\right)^2,$$

where $\Delta S_{overnight}$ is the change in the BAX price between the close of the previous day and the opening of the current day, and S_i is the intraday price (either trade or quote) observed at a certain interval (e.g. every 5 min. or every 5 tick).

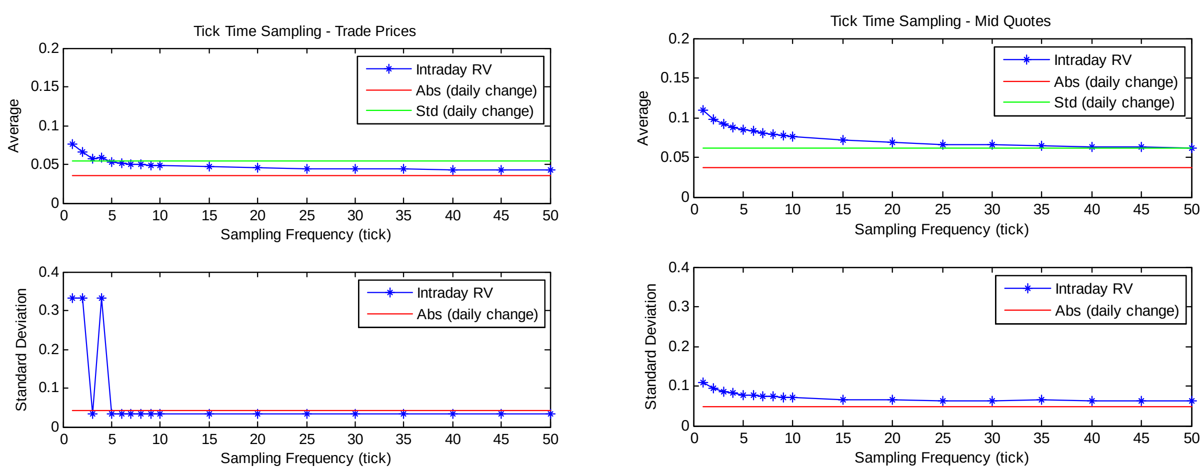
In order to choose an optimal sampling frequency, we plot the volatility signature plots (Fang (1996) and Andersen, Bollerslev, Diebold, and Labys (2000)) under four settings:

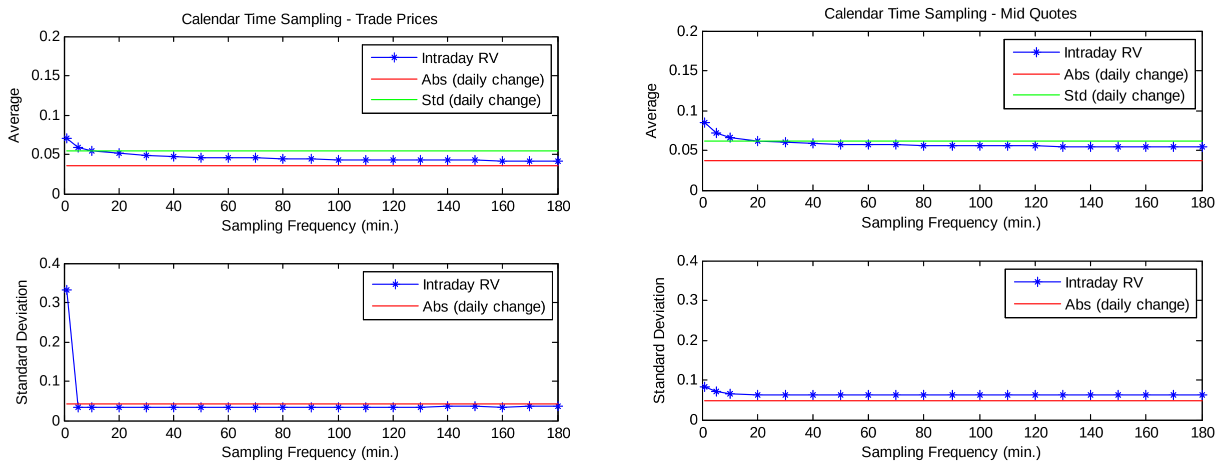
1. Tick time sampling using trade prices
2. Tick time sampling using mid-quotes
3. Calendar time sampling using trade prices
4. Calendar time sampling using mid-quotes

A volatility signature plot shows the average level of RVs at different sampling frequencies. Typically, a volatility signature plot shows a much higher level of RVs at very short sampling intervals, which decreases and converges to a certain level as the sampling interval is lengthened. This upward bias in RV when sampled very frequently is known to be due to microstructure noise. A volatility signature plot tells us beyond which sampling frequency this bias becomes negligible

The volatility signature plots for the BAX RV under four different settings are shown in the top graph of each panel in <Figure 11>. An unbiased estimate of RV will have an average value that is comparable to the standard deviation of the daily change in BAX (green). We also plot the mean of the daily absolute change in BAX (red) since the absolute value of daily change in BAX is often used to gauge the surprise in the interest rate expectation in the absence of intraday price data. The absolute value of daily change in BAX can be also interpreted as the RV in the asymptotic limit since it is equivalent to RV when the sampling is done only once a day.

In three of the volatility signature plots (all except tts-quote), we find that the plots flatten and coincide with the standard deviation of the daily change at around the 5-tick interval and the 20-min. interval, indicating that most of the microstructure noise is removed at these sampling frequencies. Only in the case of tts-quote, RV is consistently higher than the standard deviation of the daily price change up to the sampling interval of 40-ticks.





<Figure 11. Volatility signature>

We also look at how the volatility of the RVs change with sampling frequency. We would expect the RV to be noisier when it's sampled less frequently. The bottom graph of each panel shows the volatility measured by the standard deviation of daily RVs at different frequencies. Again, we add the volatility of the daily absolute price change for comparison. We find that the volatility of the RVs at very high sampling frequency is higher than the volatility of the RVs sampled at low frequency. The volatility of the RVs stay more or less constant beyond certain sampling frequencies (5-tick for tts-trade, 10-tick for tts-quote, 5 min. for cts-trade, and 10 min. for cts-quote).

One puzzling observation is that the volatility of the RV is smaller than the volatility of the daily absolute change when trade prices are used (the blue line is below the red in the LEFT panels), but it is larger when quotes are used (the blue line is above the red in the RIGHT panels). This indicates that the RV is less noisy when trade prices are used. Overall, we conclude that the RV computation using trade prices with sampling every 5-ticks yields the best results.

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Table 1. Liquidity of BAX and OBX

	BAX (average daily)				OBX (average daily)			
	All Dates		FADs		All Dates		FADs	
Year	Volume (x1000)	# Trades	Volume (x1000)	# Trades	Volume (x1000)	# Trades	Volume (x1000)	# Trades
2005	36	995	57	1478	2	4	3	5
2006	52	1120	100	1784	3	5	3	6
2007	48	1303	74	1605	3	5	5	7
2008	31	1478	48	1998	1	3	1	3
2009	24	1084	38	1595	0	0	0	0
2010	44	2125	89	3499	2	4	10	9
2011	74	3454	123	5173	2	6	6	11
2012	91	3969	130	4951				
2013	98	3836	150	5773				
all years	44	1660	71	2379	2	4	4	6

Table 2. Average daily number of OBX options by maturity

year	all maturities	< 1 month	1-3 months	3-6 months	> 6 months
2006	17	2	4	7	4
2007	26	2	6	11	7
2008	21	2	7	8	3
2010	23	2	5	9	7
2011	29	2	5	10	12
2012	24	2	5	9	9
2013	23	1	2	6	15

Table 3. Change in IV and RV around the Bank of Canada target rate announcements

For each event day, $i \in [-5, +10]$, which denotes the number of business days from a Fixed Announcement Day (FAD), we run the following regression.

$$\Delta IV_t = \alpha_i + \beta_i \cdot \mathrm{I}_i(t) + \varepsilon_{it},$$

$$\Delta RV_t = \alpha_i + \beta_i \cdot \mathrm{I}_i(t) + \varepsilon_{it},$$

where ΔIV_t and ΔRV_t are daily changes in implied volatility and realized volatility of BAX, and $\mathrm{I}_i(t)$ is an indicator function that yields 1 if date *t* is $i \in [-5, +10]$ business days away from a FAD and 0 otherwise. T-statistics that are significant at 90% confidence level are highlighted in bold red. The unit is percent (e.g. -0.07 is – 7 bps).

# business days	Daily change in IV				Daily change in RV			
from FAD	α_i	t-stat.	β_i	t-stat.	α_i	t-stat.	β_i	t-stat.
-5	0.003	(0.438)	0.020	(0.700)	0.002	(0.293)	-0.026	(-0.663)
-4	0.003	(0.446)	-0.029	(-1.219)	0.002	(0.292)	-0.003	(-0.080)
-3	0.003	(0.447)	0.025	(1.047)	0.002	(0.290)	0.029	(0.878)
-2	0.003	(0.435)	-0.017	(-0.629)	0.002	(0.287)	-0.012	(-0.361)
-1	0.003	(0.453)	-0.006	(-0.258)	0.002	(0.296)	-0.002	(-0.053)
0	0.003	(0.424)	-0.074	**(-2.913)**	0.002	(0.280)	-0.006	(-0.182)
1	0.003	(0.442)	0.002	(0.097)	0.002	(0.292)	-0.061	**(-1.838)**
2	0.003	(0.447)	-0.052	**(-2.133)**	0.002	(0.287)	0.034	(1.015)
3	0.003	(0.452)	-0.021	(-0.796)	0.002	(0.290)	-0.055	**(-1.686)**
4	0.003	(0.455)	-0.005	(-0.205)	0.002	(0.295)	-0.028	(-0.855)
5	0.003	(0.445)	-0.029	(-1.153)	0.002	(0.288)	0.030	(0.884)
6	0.003	(0.446)	0.021	(0.840)	0.002	(0.290)	0.010	(0.289)
7	0.003	(0.440)	-0.003	(-0.099)	0.002	(0.291)	-0.051	(-1.545)
8	0.003	(0.434)	0.016	(0.572)	0.002	(0.292)	0.004	(0.122)
9	0.003	(0.446)	0.001	(0.023)	0.002	(0.293)	-0.011	(-0.336)
10	0.003	(0.443)	0.009	(0.350)	0.002	(0.283)	-0.040	(-1.143)

Table 4. Important Bank of Canada policy actions between 2007 and 2012

Date	Event description	Daily change in IV (bps)
PANEL A: Crisis related policy actions, 2007-2010		
2007-Dec-04	(Not a FAD) Term PRAs announced for liquidity purposes (along with the Bank of England, the European Central Bank, the Federal Reserve, and the Swiss National Bank)	-12
2008-Mar-11	(Not a FAD) Term PRAs announced for liquidity purposes, coordinated with other G10 central banks	-25
2008-Oct-08	(Not a FAD) Unscheduled cut in target rate coordinated with other central banks	-51
2010-Apr-20	(FAD) Removal of conditional commitment/forward guidance	-18
PANEL B: Change in monetary policy bias, 2011-2012		
2011-May-31	(FAD) Tightening bias is introduced.	-10
2011-Sep-07	(FAD) Tightening bias is removed.	-26
2012-Apr-17	(FAD) Tightening bias is reintroduced.	-10
2012-Oct-23	(FAD) Tightening bias is pushed out.	-14
	Average - all days	0
	Average - FADs	-5
	Average - non FADs	1

Table 5. Forecasting realized uncertainty around the Bank of Canada target rates

The results of following univariate regressions are reported in Panel A.

$$RU_{OIS}(t, t+n) = \alpha + \beta \cdot IV_t + \varepsilon_t$$

$$RU_{OIS}(t, t+n) = \alpha + \beta \cdot RV_t + \varepsilon_t,$$

where $RU_{OIS}(t, t+n)$ is the absolute value of the prediction error of n-month OIS contract observed on the last day of month t, and IV_t and RV_t are implied volatility and realized volatility of BAX observed on the last day of month t. The maturity of RV is matched with forecasting horizon of the regression. T-statistics that are significant at 90% confidence level are highlighted in bold red. The results of the bivariate regressions are reported in Panel B.

Regressor		Forecast Horizon (n-month)						
		1	2	3	4	5	6	9
PANEL A: Univariate regression								
IV	β	0.22	0.42	0.68	0.88	1.15	1.35	1.77
	t-stat.	**(2.26)**	**(3.61)**	**(4.25)**	**(4.09)**	**(4.01)**	**(3.68)**	**(2.91)**
	adj. R^2	0.08	0.20	0.26	0.24	0.24	0.20	0.13
RV	β	0.96	1.51	2.77	3.91	4.64	5.10	8.20
	t-stat.	**(3.62)**	**(4.64)**	**(6.14)**	**(5.69)**	**(4.52)**	**(3.68)**	**(4.27)**
	adj. R^2	0.18	0.28	0.40	0.37	0.26	0.19	0.24
PANEL B: Multivariate regression								
Constant	α	-0.01	-0.33	-1.07	-2.03	-2.60	-2.50	-3.42
	t-stat.	(-0.03)	(-1.09)	**(-2.41)**	**(-2.79)**	**(-2.29)**	(-1.60)	(-1.55)
IV	β_{IV}	0.11	0.24	0.28	0.31	0.54	0.77	0.86
	t-stat.	(1.06)	**(1.94)**	**(1.74)**	(1.34)	**(1.66)**	**(1.83)**	(1.38)
RV	β_{RV}	0.77	1.16	2.38	3.52	4.04	4.13	7.12
	t-stat.	**(2.58)**	**(3.10)**	**(4.38)**	**(4.15)**	**(3.23)**	**(2.47)**	**(3.19)**
	adj. R^2	0.17	0.32	0.46	0.43	0.36	0.28	0.27

Table 6. Correlations

This table reports the correlations of monthly series of the variables between January 2006 and February 2013 excluding the period between November 2008 and March 2010 when BAX IV could not be computed. IV and RV are implied volatility and 3-month maturity realized volatility of BAX observed on the last day of the month. CBS is the spread between BBB-rated 10-year corporate bonds and the 10-year Treasury yield. TYS is the spread between 2-year and 5-year Treasury yields. EG is the employment growth. PU is the index of Canadian economic policy uncertainty published by Baker, Bloom, and Davis.

	IV	RV	CBS	TYS	EG	PU
IV	1.00	0.51	0.71	0.20	-0.60	0.50
RV		1.00	0.56	0.16	-0.14	0.38
CBS			1.00	0.65	-0.61	0.81
TYS				1.00	-0.32	0.82
EG					1.00	-0.54
PU						1.00

Table 7. Forecasting OIS excess returns, univariate regressions

We report the results of following univariate regressions.

$$R_{OIS}(t, t+n) = \alpha + \beta \cdot X_t + \varepsilon_t$$

where $R_{OIS}(t, t+n)$ is the prediction error of n-month OIS contract observed on the last day of month t. X_t denotes each of the variables listed in the table below observed on the last day of month t. The maturity of RV is matched with forecasting horizon of the regression. T-statistics that are significant at 90% confidence level are highlighted in bold red.

	OIS Maturity	1	2	3	4	5	6	9
IV	β	0.00	0.00	0.01	0.02	0.03	0.03	0.08
	t-stat.	(-0.36)	(0.23)	(1.02)	(1.16)	(1.17)	(1.30)	**(2.12)**
	adj. R^2	-0.02	-0.02	0.00	0.01	0.01	0.01	0.07
RV	β	0.02	0.06	0.11	0.16	0.21	0.28	0.54
	t-stat.	(1.33)	**(2.26)**	**(2.94)**	**(2.82)**	**(2.80)**	**(2.95)**	**(4.69)**
	adj. R^2	0.01	0.07	0.12	0.11	0.11	0.12	0.28
Corporate bond yield spread	β	0.01	0.03	0.06	0.08	0.10	0.12	0.17
	t-stat.	(0.80)	**(2.17)**	**(3.23)**	**(3.39)**	**(3.50)**	**(3.52)**	**(3.41)**
	adj. R^2	-0.01	0.06	0.15	0.16	0.17	0.17	0.16
Treasury yield term spread	β	0.01	0.02	0.03	0.04	0.05	0.06	0.06
	t-stat.	(0.92)	(1.42)	(1.40)	(1.17)	(1.18)	(1.11)	(0.83)
	adj. R^2	0.00	0.02	0.02	0.01	0.01	0.00	-0.01
Employment growth	β	0.01	-0.03	-0.05	-0.08	-0.09	-0.11	-0.12
	t-stat.	(0.28)	(-0.58)	(-0.72)	(-0.90)	(-0.81)	(-0.78)	(-0.60)
	adj. R^2	-0.02	-0.01	-0.01	0.00	-0.01	-0.01	-0.01
Policy uncertainty	β	0.02	0.06	0.09	0.12	0.15	0.18	0.18
	t-stat.	(0.92)	**(2.29)**	**(2.67)**	**(2.63)**	**(2.62)**	**(2.43)**	(1.65)
	adj. R^2	0.00	0.07	0.10	0.10	0.10	0.08	0.03
$R_{OIS}(t-3, t)$	β	-0.01	0.00	0.01	0.02	0.02	0.03	0.05
	t-stat.	(-1.88)	(-0.48)	(1.00)	(1.30)	(1.49)	(1.55)	(1.69)
	adj. R^2	0.04	-0.01	0.00	0.01	0.02	0.03	0.03
$RU_{OIS}(t-3, t)$	β	-0.01	-0.01	0.00	-0.01	-0.01	-0.01	-0.02
	t-stat.	(-1.41)	(-1.11)	(-0.41)	(-0.72)	(-0.79)	(-0.85)	(-0.75)
	adj. R^2	0.02	0.00	-0.02	-0.01	-0.01	-0.01	-0.01

Table 8. Forecasting OIS excess returns, multivariate regression

We report the results of following multivariate regressions.

$$R_{OIS}(t, t+n) = \alpha + \overline{X_t} \cdot \bar{\beta} + \varepsilon_t$$

where $R_{OIS}(t, t+n)$ is the prediction error of n-month OIS contract observed on the last day of month t. $\overline{X_t}$is a 1×8 vector containing the variables listed in the table below observed on the last day of month t. $\bar{\beta}$ is an 8×1 vector of slope coefficients. The maturity of RV is matched with forecasting horizon of the regression. T-statistics that are significant at 90% confidence level are highlighted in bold red.

	OIS Maturity	1	2	3	4	5	6	9
Constant	α	-0.04	-0.12	-0.20	-0.24	-0.29	-0.30	-0.21
	t-stat.	(-1.21)	**(-2.89)**	**(-3.41)**	**(-3.35)**	**(-3.41)**	**(-3.07)**	(-1.91)
IV	β	-0.01	-0.02	-0.04	-0.06	-0.10	-0.13	-0.19
	t-stat.	(-0.83)	(-1.53)	**(-2.00)**	**(-2.70)**	**(-3.66)**	**(-4.43)**	**(-5.32)**
RV	β	0.03	0.05	0.05	0.06	0.07	0.07	0.10
	t-stat.	(1.41)	(1.61)	(1.11)	(0.91)	(0.85)	(0.76)	(0.94)
Corporate bond yield spread	β	0.06	0.10	0.20	0.33	0.49	0.67	1.12
	t-stat.	(1.67)	**(2.15)**	**(3.14)**	**(4.49)**	**(5.73)**	**(7.20)**	**(9.79)**
Treasury yield term spread	β	-0.03	-0.08	-0.15	-0.22	-0.32	-0.41	-0.55
	t-stat.	(-1.02)	**(-2.19)**	**(-2.87)**	**(-3.63)**	**(-4.53)**	**(-5.22)**	**(-5.68)**
Employment growth	β	0.04	0.09	0.18	0.19	0.23	0.22	0.06
	t-stat.	(0.81)	(1.32)	(1.92)	(1.81)	(1.91)	(1.65)	(0.38)
Policy uncertainty	β	0.01	0.09	0.11	0.10	0.10	0.05	-0.22
	t-stat.	(0.29)	(1.58)	(1.47)	(1.10)	(1.03)	(0.48)	(-1.53)
$R_{OIS}(t-3, t)$	β	-0.01	-0.01	-0.02	-0.03	-0.04	-0.05	-0.05
	t-stat.	(-1.21)	(-1.41)	(-1.63)	**(-2.62)**	**(-2.83)**	**(-3.02)**	**(-2.40)**
$RU_{OIS}(t-3, t)$	β	-0.03	-0.02	-0.01	-0.02	-0.01	-0.02	-0.03
	t-stat.	**(-2.80)**	(-1.51)	(-0.52)	(-0.73)	(-0.60)	(-0.80)	(-0.87)
adj. R^2 [all predictors]		0.18	0.31	0.40	0.55	0.65	0.72	0.81
adj. R^2 [exclude IV and RV]		0.17	0.26	0.36	0.49	0.55	0.60	0.69