On the use of leverage caps in bank regulation

Afrasiab Mirza*
Department of Economics
University of Birmingham
a.mirza@bham.ac.uk

Frank Strobel
Department of Economics
University of Birmingham
f.strobel@bham.ac.uk

April 1, 2013

Abstract

This paper demonstrates that a flat capital charge in conjunction with a fixed leverage ratio restriction can be as effective in curtailing socially inefficient risk-taking by banks as risk-weighted capital rules. We consider a banking sector that consists of a single bank acting as a monopolist in the loan market, and financed through debt and equity. The bank has incentives to take inefficient risks because it doesn't internalize the social costs stemming from bank failure (due to limited liability) and because it's debt is subsidized (e.g. because of mis-priced deposit insurance). In this framework, we find that the regulator can implement the socially optimal allocation of risk using either risk-weighted capital regulation or a combination of a risk-independent capital rule together with a leverage restriction. As it is easier in practice to measure leverage rather than establish the correct risk-weights, we conclude that leverage caps may be an effective substitute for risk-weighted capital requirements.

JEL Classification Codes: G21,G32, G38

Keywords: capital regulation, leverage caps, moral hazard, too-big-to-fail

*Corresponding Author.

We thank Laetitia LePetit for valuable comments and suggestions.

1 Introduction

Following the financial crisis of 2008-2009, policy makers have sought to tighten regulations governing the financial sector. Specifically, the new Basel III international standards on banking regulation have introduced a number of new regulatory instruments to complement traditional capital regulation. One such additional instrument is a non-risk based leverage ratio restriction. While a large literature exists on various aspects of capital regulation (see for example the recent survey by **?**), relatively little is known about the implications of using leverage restrictions for the purpose of prudential regulation.

This paper examines the use leverage ratio restrictions in mitigating risk-taking behaviour by banks. We find that when banks take too much risk, either because they do not internalize the socials costs of failure or because of perverse incentives stemming from government guarantees of their debt, leverage caps are effective *substitutes* for risk-weight capital requirements in curbing excessive risk-taking. Given that risk-based capital weights are difficult to implement in practice because they can be manipulated by banks, this result implies that the use of leverage caps may help regulators overcome such manipulation.

We examine a single bank operated by management in the interest of its owners, abstracting from conflicts between bank managers and owners. The bank raises funds from outside investors in the form of capital and debt and can invest in risky loans. The failure of the bank entails negative social externalities that are not internalized by its owners as they enjoy limited liability. Externalities can be mitigated by increasing bank capital. However, the bank does not have incentives to hold the socially efficient level of capital as all costs of failure are not borne by the bank, thus motivating the need for regulation. The regulator has recourse to two sets of regulatory instruments: risk-weighted capital requirements and a flat capital requirement combined with leverage restrictions. The key contribution of the paper is to show that both sets of instruments can induce banks to hold the socially optimal level of capital and take the socially optimal level of risk.

This paper is most closely related to **?** who also studies the use of leverage restrictions in prudential regulation but in a framework of adverse selection. **?** examines how leverage restrictions, together with risk-weighted capital requirements, can induce banks to truthfully reveal private information on bank risk. Thus, in the presence of asymmetric information between regulators and the bank on the nature of portfolio risk, leverage restrictions can be seen as complimenting standard risk-weighted capital requirements. On the other hand, this paper shows that in the presence of moral hazard, when regulators cannot dictate portfolio decisions by bank managers, leverage restrictions can substitute for risk-weighted capital requirements.

This paper also contributes to the broader literature on the use of leverage as an additional instrument in the prudential supervisory toolkit. Following **?**, there is increasing evidence that leverage is an important variable in understanding balance sheet adjustments for banks. Furthermore, **?** argue that information regarding bank leverage is also more directly observable in comparison with portfolio allocations. As a result, the literature has argued for the need to reduce excessive bank leverage. For instance, **?** shows that limiting bank leverage is necessary to prevent asset bubbles but does not examine the substitutability between risk-weighted capital requirements and leverage restrictions. **?** considers both liquidity and solvency risks and shows leverage restrictions are needed to control solvency risk but does not consider risk-weighted capital requirements.

The rest of paper is organized as follows. The next section presents the model followed by an analysis of effectiveness of leverage restrictions. We then extend the model to incorporate government guarantees of bank debt before concluding in the last section.

2 Model

We consider a static model with three principle actors: investors/depositors, a bank, and a regulator.

2.1 Investors

Investors are risk-neutral and may invest in risk-free assets or risky debt issued by the bank. The gross return on risk-free assets is R_f. Investors are willing to invest in bank debt as long as they break-even.

2.2 Bank

The banking sector consists of a single bank acting as a monopolist in the loan market. The bank is operated by its owners and is risk-neutral. To finance it's operations, the bank issues a quantity of debt D at a gross interest rate of R_d. It also holds an amount of capital K that we take to be the wealth invested by the bank's owners.[1]

[1] This definition of capital is consistent with the literature on the effects of capital regulations on the portfolio choices of banks. See for instance **?**, **?** and **?**.

The cost of raising capital is increasing and convex in the amount of capital raised and is denoted by $C(\cdot)$ where $C' > R_f$ and $C'' > 0$.

The bank generates revenues by investing L in risky loans. However, it incurs operating costs $\gamma(L)$ as a result of loan-making activity. These operating costs are to be interpreted as the costs of screening and managing borrowers. Moreover, costs are presumed to be convex in the quantity of loans issued: $\gamma', \gamma'' > 0$.

The gross return on loans is R. Since loans are risky returns are stochastic with the distribution of returns being denoted by $F(R)$ with support on $[\underline{R}, \overline{R}]$. We assume that the bank takes the distribution of returns as given. Moreover, the bank finds it worthwhile to issue risky loans as the expected return on such loans exceeds the risk-free return:

Assumption 1.

$$\int_{\underline{R}}^{\overline{R}} RdF(R) \equiv R_\mu > R_f. \tag{1}$$

The bank is protected by limited liability and goes under when it can no longer afford to repay its creditors. Clearly, this occurs whenever returns on the projects financed by the risky loans is sufficiently small. Given an amount of loans L and debt D, the bank defaults whenever $RL - \gamma(L) - R_d D < 0$. In other words, defaults occurs whenever $R < R_b$ where

$$R_b = \frac{R_d D + \gamma(L)}{L} \tag{2}$$

is the break-even level of returns. Upon default, creditors seize the assets of the bank and are entitled to any residual value while the owners are wiped out. Also, note that the probability of default is:

$$F(R_b) = F\left(\frac{R_d D + \gamma(L)}{L}\right) \tag{3}$$

and obviously depends not only on $F(\cdot)$ but also on bank choices.

The bank's balance sheet matches liabilities (debt and equity) with assets (loans) so that $D + K = L$. Therefore, the bank's problem in the absence of regulation is to maximize expected profits subject to participation by investors while satisfying the balance-sheet condition:

$$\max_{D,L,K\geq 0} \pi^p = \int_{R_b}^{\overline{R}} (RL - \gamma(L) - R_d D) dF(R) - C(K) \text{ subject to}$$
$$R_f D = \int_{R_b}^{\overline{R}} R_d D dF(R) + \int_{\underline{R}}^{R_b} (RL - \gamma(L)) dF(R)$$
$$D + K = L.$$

The bank's limited liability is encapsulated in the lower limit on the integral in π^p. This ensures that the bank only values non-negative profits as R_b is the break-even level of returns. Also, creditors can expect their promised return $R_d D$ whenever the bank is solvent or whenever $R \geq R_b$. When $R < R_b$, the bank is insolvent and creditors are entitled to the residual value $RL - \gamma(L)$.

2.3 Regulator

We assume there is a regulator tasked with maximizing social welfare, W that is defined as the sum of expected bank profits minus the expected costs of bank failure. The regulator values bank profits because they capture the total surplus from socially valuable intermediation activity (recall that the bank is a monopolist in the loan market). However, the regulator is also concerned about bank failures (especially of large banks) because they impose significant negative externalities on society.[2] As in **?**, the social costs of failure may be taken to be proportional to the losses of the bank:

$$S = \Phi \int_{\underline{R}}^{R_b} (RL - \gamma(L) - R_d D)\, dF(R). \tag{4}$$

where $\Phi > 1$ captures externalities imposed on society due to bank failure.[3]

Due to limited liability, the bank's owners do not take these failure externalities into account when choosing capital, debt and investment. Nor do the bank's creditors when setting the interest rate on bank debt. As a result, the bank takes too much risk relative to the socially efficient level despite market discipline. This problem of excessive risk-taking is amplified when the regulator chooses to bail out the bank's creditors because these bailouts weaken market discipline.

[2] These may take the form of a reduction in the amount of credit available to the real economy (a credit crunch) or rapid and large-scale liquidations of bank assets that drive down prices (fire-sales). For a discussion of these externalities see **?**.

[3] Alternatively, the social costs may be a function of the total debt or total quantity of loans issued by the bank. These specifications yield qualitatively similar results.

Even without (mis-priced) debt guarantees to bank creditors, the presence of failure externalities provides a role for regulating the bank. Nevertheless, regulation must balance the need to reduce the expected costs of failure with the value of intermediation activity. Minimizing the externalities from bank failure requires that the bank hold more capital and reduce the amount of risky loans it issues. However, these loans finance socially valuable projects so excessive reductions in loan making activity are also undesirable.

The regulator can impose two types of regulations: a risk-based minimum capital requirement or a combination of a capital requirement and leverage ratio cap that are both independent of risk. In the context of the above framework, a risk-based minimum capital requirement is a capital level $\underline{K}(\cdot)$ that the bank must maintain at all times. $\underline{K}(\cdot)$ may be non-decreasing function of bank risk that is measured by the amount of risky loans L. Risk can be measured in this way because it is assumed that the regulator knows the distribution of returns $F(\cdot)$. When capital is set independent of risk $\underline{K}(\cdot) = \underline{K}$. A leverage ratio restriction is an upper bound on bank leverage $\mathcal{L} \equiv L/K$ (say $\overline{\mathcal{L}}$) that the bank cannot exceed.[4] In accordance with Basel III rules, the leverage ratio restriction is not sensitive to risk.

2.4 Timing

As for timing, the regulator first announces regulations $\{\underline{K}(\cdot), \overline{\mathcal{L}}\}$, the bank then chooses its portfolio size and composition taking into account the regulations.

3 Benchmark

We first characterize the level of capital, debt and loans that the bank chooses in the absence of government intervention. In this case, the bank maximizes expected profits subject to the participation constraint of creditors and the balance sheet condition:

$$\max_{D,L,K\geq 0} \pi^p \equiv \int_{R_b}^{\overline{R}} (RL - \gamma(L) - R_d D) dF(R) - C(K) \text{ subject to} \tag{5}$$

$$R_f D = \int_{R_b}^{\overline{R}} R_d D dF(R) + \int_{\underline{R}}^{R_b} (RL - \gamma(L)) dF(R) \tag{6}$$

$$D + K = L. \tag{7}$$

[4] Leverage is defined as the ratio of total assets, L, to capital, K. Alternative definitions of leverage such as D/L or L/K yield qualitatively similar results.

We can rewrite the bank's objective using the participation constraint of investors as follows:

$$\max_{L,K\geq 0} \pi^p = (R_\mu - R_f)\, L - \gamma(L) + R_f K - C(K) \tag{8}$$

where we have used the balance sheet condition to eliminate D. The above expression says that the bank chooses capital and loans to maximize the sum of net present value of investment and the return from capital. The bank's optimal choice of loans equates the net return from an additional unit of loans with the corresponding marginal cost:

$$R_\mu - R_f = \gamma'(L). \tag{9}$$

On the other hand, the bank's optimal choice of capital is zero because profits are strictly decreasing in capital as $C'(0) > R_f$ and $C''(0) > 0$. Thus, in equilibrium the bank raises debt, holds no capital and invests all proceeds from debt in risky loans. We summarize this in the following proposition:

Proposition 1. *Denote the optimal bank choices in the absence of regulation and debt guarantees by* (D^p, L^p, K^p). *Then, in equilibrium* $K^p = 0$, $D^p = L^p$, *and* L^p *is characterized by* (9).

When the government provides debt guarantees to creditors (as in the case of deposit insurance), it weakens the creditors' incentives to discipline the bank. In fact, if creditors are bailed out whenever the bank is insolvent, their participation constraint is then $R_f D = \int_{\underline{R}}^{\overline{R}} R_d D dF(R)$ implying that $R_d = R_f$. Thus, as government guarantees turn bank debt into a safe asset creditors no longer have the incentives to exert any discipline on bank behaviour.

With debt guarantees the bank's problem is to maximize expected profits subject to $R_d = R_f$ and the balance sheet condition:

$$\max_{D,L,K\geq 0} \pi^g \equiv \int_{R_b}^{\overline{R}} (RL - \gamma(L) - R_f D) dF(R) - C(K) \text{ s.t. and } D + K = L. \tag{10}$$

Rewriting bank profits using the balance sheet condition we obtain:

$$\max_{L,K\geq 0} \pi^g = \int_{R_b}^{\overline{R}} ((R - R_f)L - \gamma(L) + R_f K) dF(R) - C(K) \tag{11}$$

$$= \pi^p - \int_{\underline{R}}^{R_b} ((R - R_f)L - \gamma(L) + R_f K) dF(R) \tag{12}$$

where the second term in the last equation is negative and represents the expected losses in bankruptcy. Hence, government debt guarantees provide the bank with a subsidy equal to their expected losses in bankruptcy. As a result, the bank then takes on too much debt and risk. This is shown formally in the following proposition:

Proposition 2. *Denote the optimal bank choices when the regulator provides debt guarantees by* (D^g, L^g, K^g)*. Then, debt guarantees generates a standard moral hazard problem whereby banks takes on too much debt and risk:* $D^g > D^p$ *and* $L^g > L^p$*.*

Proof. The optimal level of loans is characterized by the following first-order condition:

$$\begin{aligned}&(R_\mu - R_f)\\ &\quad - \left[\int_{\underline{R}}^{R_b} ((R - R_f) - \gamma'(L))dF(R) + \frac{\partial R_b}{\partial L}((R_b - R_f)L - \gamma(L) + R_f K)\right] = \gamma'(L).\end{aligned} \tag{13}$$

where we have used Leibniz's rule as R_b is a function of L. However, using the definition of R_b, note that $(R_b - R_f)L - \gamma(L) + R_f K = 0$. Therefore, this expression can be written as:

$$(R_\mu - R_f) - \int_{\underline{R}}^{R_b} ((R - R_f) - \gamma'(L))dF(R) = \gamma'(L).$$

or more simply as

$$\frac{1}{1 - F(R_b)} \left[\int_{R_b}^{\overline{R}} (R - R_f)dF(R)\right] = \gamma'(L) \tag{14}$$

Then, as $R_b \geq \underline{R}$, the term in the square brackets is larger than the LHS of (9). Then, as $F(R_b) < 1$, the LHS is larger than the LHS of (9) for all possible L, K. Finally, as γ' is strictly increasing, this implies that $L^g > L^p$. Then, the marginal benefit of capital is:

$$\begin{aligned}R_f - &\left[\int_{\underline{R}}^{R_b} R_f dF(R) + \frac{\partial R_b}{\partial K}((R_b - R_f)L - \gamma(L) + R_f K)\right]\\ &= R_f(1 - F(R_b))\\ &\leq R_f\\ &\leq C'(0).\end{aligned}$$

Therefore, $K^g = K^p = 0$, and thus $D^g = L^g > L^p = D^p$ via the balance sheet condition. □

In the remainder of the paper we assume that the regulator provides debt guarantees.

4 The Social Optimum and Implementation

The socially optimal levels of capital, debt and loans trade-off the benefits of the intermediation activities of the bank against the social costs from bank failure. Balancing these different concerns is even more important with debt guarantees as these induce the bank to issue more loans but also raise the expected costs of failure.

4.1 Socially Optimal Level of Risk

Given these guarantees, the regulator problem is to maximize social welfare subject to $R_d = R_f$ and the balance sheet condition:

$$\max_{D,K,L\geq 0} W = \int_{R_b}^{\overline{R}} (RL - \gamma(L) - R_f D) dF(R) - C(K) \\ + \Phi \int_{\underline{R}}^{R_b} (RL - \gamma(L) - R_f D)\, dF(R) \text{ s.t. } D + K = L \quad (15)$$

We can rewrite the regulator's problem using the balance sheet condition as follows:

$$\max_{L,K\geq 0} W = \int_{\underline{R}}^{\overline{R}} ((R - R_f)L - \gamma(L) + R_f K) dF(R) - C(K) \\ + (\Phi - 1) \int_{\underline{R}}^{R_b} ((R - R_f)L - \gamma(L) + R_f K) dF(R) \quad (16)$$

As is clear from the above expression, the regulator balances the total surplus from intermediation activity against the expected costs of bank failure.

The socially optimal choices of debt and capital are characterized by the following first-order conditions:

$$L: \ (R_\mu - R_f) + (\Phi - 1) \left[\int_{\underline{R}}^{R_b} ((R - R_f) - \gamma'(L)) dF(R) \right] = \gamma'(L) \quad (17)$$

$$K: R_f + (\Phi - 1) \left[\int_{\underline{R}}^{R_b} R_f dF(R) \right] = C' \quad (18)$$

where we have used $(R_b - R_f)L - \gamma(L) + R_f K) = 0$ to simplify the expressions. To facilitate comparison with the benchmark cases, we can rearrange these expressions as follow:

$$L: (R_\mu - R_f) = \frac{(\Phi - 1)}{1 + (\Phi - 1)F(R_b)} \left[\int_{\underline{R}}^{R_b} (R_\mu - R) dF(R) \right] + \gamma'(L) \tag{19}$$

$$K: \quad R_f \left[1 + (\Phi - 1)F(R_b)\right] = C' \tag{20}$$

Equation (19) implies that the socially optimal level of loans is chosen to equate the marginal social benefit of intermediation with the corresponding marginal social cost. The marginal social benefit is just net expected returns which are also the private marginal benefit. However, the marginal social cost now includes not only the cost of operations but also the expected marginal cost of failure (first term on the LHS). We show in the proposition below that this is positive and therefore the socially optimal choice of loans requires a reduction below the level in either benchmark scenario. Moreover, by examining (20), we see that the optimal level of capital is positive whenever Φ is sufficiently large. Intuitively, this is because capital is socially valuable: it lowers the social costs of bank failure by acting as a cushion for bank losses. We summarize these results in the following proposition:

Proposition 3. *Denote the socially optimal choices of debt, loans and capital by* (D^*, L^*, K^*). *Then, to take into account the externalities due to bank failure, and the moral hazard due to debt guarantees the bank must borrow and lend less and hold more capital:* $D^* < D^p < D^g$, $L^* < L^p < L^g$ *and* $K^* > K^p = K^g$.

Proof. □

4.2 Implementation via Risk-Based Capital Regulation

The regulator can implement the optimal allocation through a standard risk-based minimum capital requirement. Alternatively, the regulator can also implement the optimal allocation through a combination of two non-risk based instruments: a fixed minimum capital requirement and a fixed leverage ratio restriction. Therefore, the regulator can implement the optimal allocation even when risk levels in the bank are not directly observable or verifiable.

We first consider implementation via a risk-based minimum capital requirement. Consider an arbitrary minimum capital rule $\underline{K}(L)$ that is strictly increasing (and differentiable) in the amount of risky loans issued by the bank. Given such a rule, the bank's problem is as follows:

$$\max_{D,L,K\geq 0} \pi^r \equiv \int_{R_b}^{\overline{R}} (RL - \gamma(L) - R_f D) dF(R) - C(K) \text{ s.t.}$$
$$D + K = L$$
$$K \geq \underline{K}(L)$$

We can write the bank's problem as follows:

$$\max_{L,K\geq 0} \pi^r = \int_{R_b}^{\overline{R}} ((R - R_f)L - \gamma(L) + R_f K) dF(R) - C(K)$$
$$K \geq \underline{K}(L)$$

As the bank's profits are strictly decreasing in capital, the minimum capital requirement will be binding in equilibrium. Therefore, if the regulator chooses $\underline{K}(L^*) = K^*$, the socially optimal choice of capital can be implemented. As a result, the optimal level of debt is characterized by the following first-order condition:

$$(R_\mu - R_f) - \int_{\underline{R}}^{R_b} ((R - R_f) - \gamma'(L)) dF(R) + R_f \underline{K}'(1 - F(R_b)) = \gamma'(L). \tag{21}$$

Then, if $\underline{K}'(L^*)$ is chosen carefully, it is possible to implement the socially optimal choice of debt and loans:

Proposition 4. *A minimum capital rule $\underline{K}(L)$ such that $\underline{K}(L^*) = K^*$ and $\underline{K}'(L^*) = \left(\frac{(\Phi-1)}{1+(\Phi-1)F(R_b^*)} - \frac{F(R_b^*)}{1-F(R_b^*)}\right)\left[\int_{\underline{R}}^{R_b^*}(R_\mu - R)dF(R)\right]$ can implement the socially optimal choices (D^*, L^*, K^*).*

Proof. Let the minimum capital rule be given by $\underline{K}(L) = \begin{cases} m \cdot (L - L^*) + K^* & L \leq L^* \\ L^* & L > L^* \end{cases}$ where $m = \frac{1}{R_f}\left(\frac{(1-\Phi)}{1+(\Phi-1)F(R_b^*)} - \frac{1}{1-F(R_b^*)}\right)\left[\int_{\underline{R}}^{R_b^*}(R_\mu - R)dF(R)\right]$. Then, given the regulations, the bank's problem is:

$$\max_{D,K\geq 0} \int_{R_b}^{\overline{R}} ((R - R_f)L - \gamma(L) + R_f \underline{K}(L)) dF(R) - C(K)$$

where we have used the fact that the minimal capital requirement will be bind because the bank's profits are decreasing in capital and $\underline{K}(L) \geq 0$ for all L. Then, the optimal choice of debt is given by:

$$
\begin{aligned}
(R_\mu - R_f) &= \frac{-1}{1-F(R_b)}\left[\int_{\underline{R}}^{R_b}(R_\mu - R)dF(R)\right] - R_f m + \gamma'(L) \\
&= \frac{(\Phi-1)}{1+(\Phi-1)F(R_b^*)}\left[\int_{\underline{R}}^{R_b^*}(R_\mu - R)dF(R)\right] + \gamma'(L)
\end{aligned}
$$

Then, given that $\gamma(L)$ is strictly convex and the equation above is equivalent to (19), the optimal choice of debt is indeed L^*. Then, given that $\underline{K}(L^*) = K^*$, the equilibrium choice of capital is in fact K^*. Finally, $D^* = L^* - K^*$ via the balance sheet condition so the socially optimal choices can indeed be implemented.

□

The key to the above result is that the choice of capital doesn't affect the bank's choice of loans on the margin. This separability ensures that the level and slope of the minimum capital requirement are sufficient to implement the optimal levels of capital and loans. In fact, we can exploit this separability to implement the social optimum without risk-based instruments.

4.3 Implementation with Leverage Caps

When the regulator specifies a flat minimum capital requirement $\underline{K}$ together with a leverage constraint $\overline{\mathcal{L}}$, the bank's problem is:

$$
\begin{aligned}
\max_{D,L,K\geq 0} \pi^r \equiv \int_{R_b}^{\overline{R}}&(RL - \gamma(L) - R_f D)dF(R) - C(K) \text{ s.t.} \\
D + K &= L \\
K &\geq \underline{K}(L) \\
L/K &\leq \overline{\mathcal{L}}
\end{aligned}
$$

We can write the bank's problem as follows:

$$
\begin{aligned}
\max_{L,K\geq 0} \pi^r &= \int_{R_b}^{\overline{R}}((R-R_f)L - \gamma(L) + R_f K)dF(R) - C(K) \\
K &\geq \underline{K} \\
L/K &\leq \overline{\mathcal{L}}
\end{aligned}
$$

As the bank's profits are strictly decreasing in capital, it is obvious that the minimum capital requirement will be binding, thereby reducing the bank's problem to:

$$\max_{L,K\geq 0} \pi^r = \int_{R_b}^{\overline{R}} ((R - R_f)L - \gamma(L) + R_f \underline{K}) dF(R) - C(\underline{K})$$
$$L/\underline{K} \leq \overline{\mathcal{L}}$$

Now, since the level of $\underline{K}$ does not affect the marginal choice of L, the bank will attempt to choose the same level of debt as in the private market case, unless the leverage constraint binds. If the latter is chosen carefully, then in equilibrium, the leverage restriction will limit bank debt and the bank's investment in risky loans. In fact, to implement the socially optimal level of loans, the regulator can simply choose $\underline{K} = K^*$ and $\overline{\mathcal{L}} = \underline{K}/L^* = K^*/L^*$.

Proposition 5. *When the regulator selects the following rules* $\{\underline{K} = K^*, \overline{\mathcal{L}} = L^*/K^*\}$*, the socially optimal levels of capital, debt and loans are implemented.*

Proof. Note that we can also write the leverage restriction as follows $\mathcal{L} = L/K \leq \overline{\mathcal{L}}$. This is equivalent to $L \leq \overline{\mathcal{L}} K = (L^*/K^*)K$ under the regulator's rules. The bank's problem is equivalent to :

$$\max_{L,K\geq 0} \pi^r = \int_{R_b}^{\overline{R}} ((R - R_f)L - \gamma(L) + R_f K) dF(R) - C(K)$$
$$K \geq K^*$$
$$L \leq (L^*/K^*)K^* = L^*$$

We know that the first constraint binds in equilibrium as the bank's profits are decreasing in capital. Hence, the equilibrium level of capital is socially optimal. Then, the optimal choice of debt solves $\frac{1}{1-F(R_b)} \left[\int_{R_b}^{\overline{R}} (R - R_f) dF(R) \right] = \gamma'(L)$. We know from Proposition 3 that this characterizes L^g which is larger than L^*. Therefore, the second constraint also bind in equilibrium implying that the optimal choice of debt is L^*. Then, via the balance sheet condition the optimal level of debt is indeed $D^* = L^* - K^*$. □

The intuition for this result is straight-forward. When risk-based capital requirements are infeasible, a fixed level of capital and a leverage cap can indirectly control risk because $L = K \cdot \mathcal{L}$. Hence, in this simple environment it is possible to implement the socially optimal outcome without resorting to risk-weighted capital charges as long as the regulator can impose leverage caps.

5 Discussion

We have shown that leverage constraints are an effective tool for regulators to limit bank size as these restrictions implicitly limit the bank's ability to accumulate risky assets. This is quite different from **?** where bank size is fixed and leverage restrictions are used as an indirect mechanism to boost bank capital. Moreover, we have also shown that if failure externalities are proportional to losses (as is likely), then such restrictions may be desirable from a social standpoint. In other words, regulators can and should use leverage restrictions to prevent banks from becoming too-big-to-fail ex-ante.

Nevertheless, the imposition of such restrictions requires accurate information about the riskiness of the bank's assets. As is well known, acquiring such information is difficult in practice. Therefore, an important aspect that remains to be explored is how effective leverage restrictions are in controlling risk-taking behaviour when banks have private information about the riskiness of their assets. In fact, it is very likely that incentivizing banks to truthfully report such information would require relaxing the minimum capital and/or leverage requirements.

Furthermore, the recent crisis has also highlighted another source of informational asymmetry between banks and regulators: the balance sheet. Through shadow banking, banks were able to effectively hide the size and scope of their asset holdings. In other words, bank choices (D, L, K) may not be perfectly observable by regulators. That would severely limit the ability to implement stringent regulation. In the presence of such severe informational constraints, additional instruments are necessary to provide banks with the correct incentives. **?** shows that if the regulator can credibly commit ex-ante to not bailing out the bank (or only with a low probability), then this can serve to reduce risk-taking by banks. Moreover, this sort of instrument does not depend on detailed information on bank risks or balance sheets.

A substitute for direct regulation is course market discipline. In our framework, creditors are able to effectively limit bank risk through higher interest rates on bank debt. However, as we have shown, government debt guarantees can erode such discipline. When the regulator has complete information on bank risks and bank balance sheets this erosion of discipline can be perfectly compensated for through tighter capital and leverage restrictions. In the absence of complete information, weaker market discipline can harm the regulator's ability to gather information via prices on bank debt or equity issues because investors may not have the incentives to produce such information. Hence, weak market discipline may even exacerbate the informational asymmetries discussed above.

We leave the formal analysis of the impact of informational asymmetries on the ability of the regulator to implement the socially optimal outcomes for future work.

6 Conclusion

In this paper, we have examined the use of leverage caps in prudential regulation. We have found that if they are used in conjunction with flat capital requirements, they can be as effective as risk-weighted capital requirements in mitigating risk-taking behaviour by banks. Moreover, as the informational requirements to implement leverage caps (and flat capital requirements) are less prone to manipulation than risk-weighted capital requirements, they may be a more effective regulatory instrument.